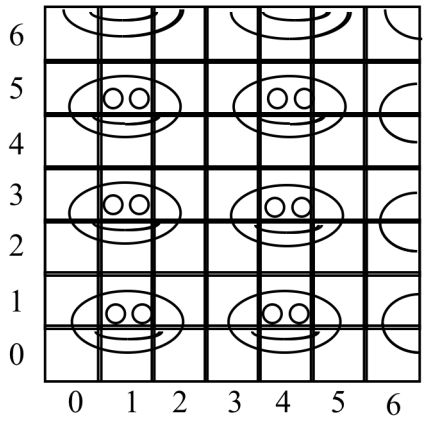
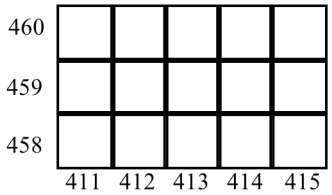
Math 44 **Sample Test Questions, Exam 1** Spring 2008
Open book, open notes, calculator allowed.

(1) Pattern problem.
If this pattern continues throughout the plane, what pictures fall in the boxes shown on the right? (Draw the pattern.)





(2) (a) Find the check digit for the ISBN 0-674-01325-__

(b) Find the check digit for the UPC code 780674 01325___

(3) Patterns with the Fibonacci numbers 1,1,2,3,5,8, etc. Give the next row of the following pattern, and complete the general formula for the "even case":

$(1)(3) = (1)(2) + 1$	$(F_1)\,(F_4) = (F_2)\,(F_3) + 1$
$(1)(5) = (2)(3) - 1$	$(F_2)\,(F_5) = (F_3)\,(F_4) - 1$
$(2)(8) = (3)(5) + 1$	$(F_3)\,(F_6) = (F_4)\,(F_5) + 1$
$(3)(13) = (5)(8) - 1$	$(F_4)\,(F_7) = (F_5)\,(F_6) - 1$
_____	_____
(Show numbers here)	(Use the F notation here.)

General pattern or formula when k is an odd number: $(F_k)\,(F_{k+3}) =$ _____

(4) (3) Circle True or False:

(a) True or False: Every integer greater than 1 is divisible by at least one prime number.

(b) True or False: The RSA code uses the one key to place a message into code and a different key to decode the coded message.

(c) True or False: 2008^{2008} is congruent to 3, mod 4.

(5) (a) Express the number 60 as the sume of two prime numbers in two different ways:

60 = ____ + ____

60 = ____ + ____

(b) This is an example of the _____ Question (or Conjecture).

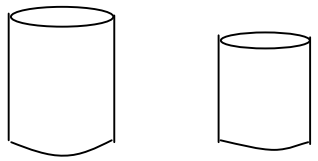
(c) If the conjecture in part b is true, explain why every odd number greater than 5 would be the sum of exactly three primes.

(d) The _____ Conjecture states that there are an infinite number of pairs of prime numbers that differ by 2.

Circle the correct answer:
(e) True or False. The ratio of two successive Fibonacci numbers is always the fraction 1/2.

(6) An antique store uses 3 kinds of packaging to ship small objects, all rectangular boxes that are all one inch deep:
4 inches by 10 inches, 5 inches by 9 inches, and 7 inches by 7 inches. They plan to ship a long, thin knife that is 10 inches long, and less than 1 inch wide at all points. Which, if any, of the boxes will it fit in – check those it will fit in:
(a) 4 by 10 Yes ____ No ____
(b) 5 by 9 Yes ____ No ____
(c) 7 by 7 Yes ____ No ____

(7) You have two measuring cups. One holds exactly 9 ounces, and the other holds exactly 4 ounces. There are no markings on the cups and you are not able to mark the cups at all. You are given a huge bucket of water. Is it possible to measure and place exactly 2 ounces of water into the larger cup? If so, carefully explain your method; if not, explain why not.



9 ounces 4 ounces

(8) We saw how the pigeonhole principle is used in surprising ways. Use it to answer the following questions:

(a) You have a drawer full of an uncoordinated jumble of a large number of red, white, blue, green, and black socks. The electricity has gone off again, it is night, and you need to pick out a pair of matched socks. What is the least number of socks you must take to guarantee that a pair of them are the same color? ____
Explain your reasoning:

(b) What is the fewest number of socks you must take out to guarantee that you have at least two pairs of matched socks (for example, two red socks and two blue socks, or four black socks)? ____

(c) If five words are chosen from this list {BAR, BAT, BUG, GAS, ROB, SAT, TAG, TOT}, then there must be a pair of those words that have the property that the last letter of one of the pair is the first letter of the other word of that pair. Use the pigeonhole principle to explain why. You might want to use these boxes as pigeonholes, properly labeled. Your explanation must be in **complete sentences**!

(9) On Monday Allen sold an old computer to Cathy for $200 and a printer to Bonnie for $50.
On Tuesday, Cathy sold the computer to Dan for $250, and Bonnie sold the printer to Dan for $60.
On Wednesday, Dan sold the computer to Bonnie for $300, and the printer to Cathy for $70.
And on Thursday, Bonnie sold the computer back to Allen for $350, and Cathy sold the printer back to Allen for $80. What was the total result? Circle made or lost for each person, and supply the amount.

(a) Allen made / lost _____

(b) Bonnie made / lost _____

(c) Cathy made / lost _____

(d) Dan made / lost _____

(10) Show at which corner points you would place the minimum number of video cameras to guard this art gallery. That number is _____.