Math 46 Sample Test 1 Fall 2008

The first test will have about ten problems, similar to ten of the problems here. We might not have covered all the material here, but will do so by the next class!
Open book, open notes, calculator allowed.

(1) [Section 2.1] Set A = {2,4,6,8,10}. Set B = {1,3,5,9}. Set C = {1,2,3,4,5}. The Universe for this problem is {1,2,3,4,5,6,7,8,9,10}.
(a) List the elements in $B \cup \overline{C}$: ________________________
(b) List the elements in $\overline{B} \cap C$: ________________________
(c) How many elements in $A \cup B$? _____

(2) [Take-away game] Suppose you play the following game with an opponent: you start with a pile of 100 bottletops. Each of you may remove 1,2,3,4,5,or 6 bottletops on each turn. The player who removes the last bottletop is the winner.
(a) Which player has a winning strategy, the one who goes first or the one who goes second?
(b) Describe the winning strategy:

(c) Fill in the first blank with one of the digits 0, 1, 2, 3, 4, 5, or 6, and fill in the second blank with the modulus that applies to this problem (and which explains the winning first move!):

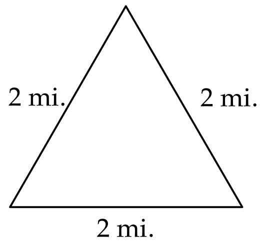
100 ≡ ____ (mod ____)

(3) [Section 1.3 – "Gauss's trick"]. Find the sum 100 + 101 + 102 + … + 200 = ________
Explain your method of solution:

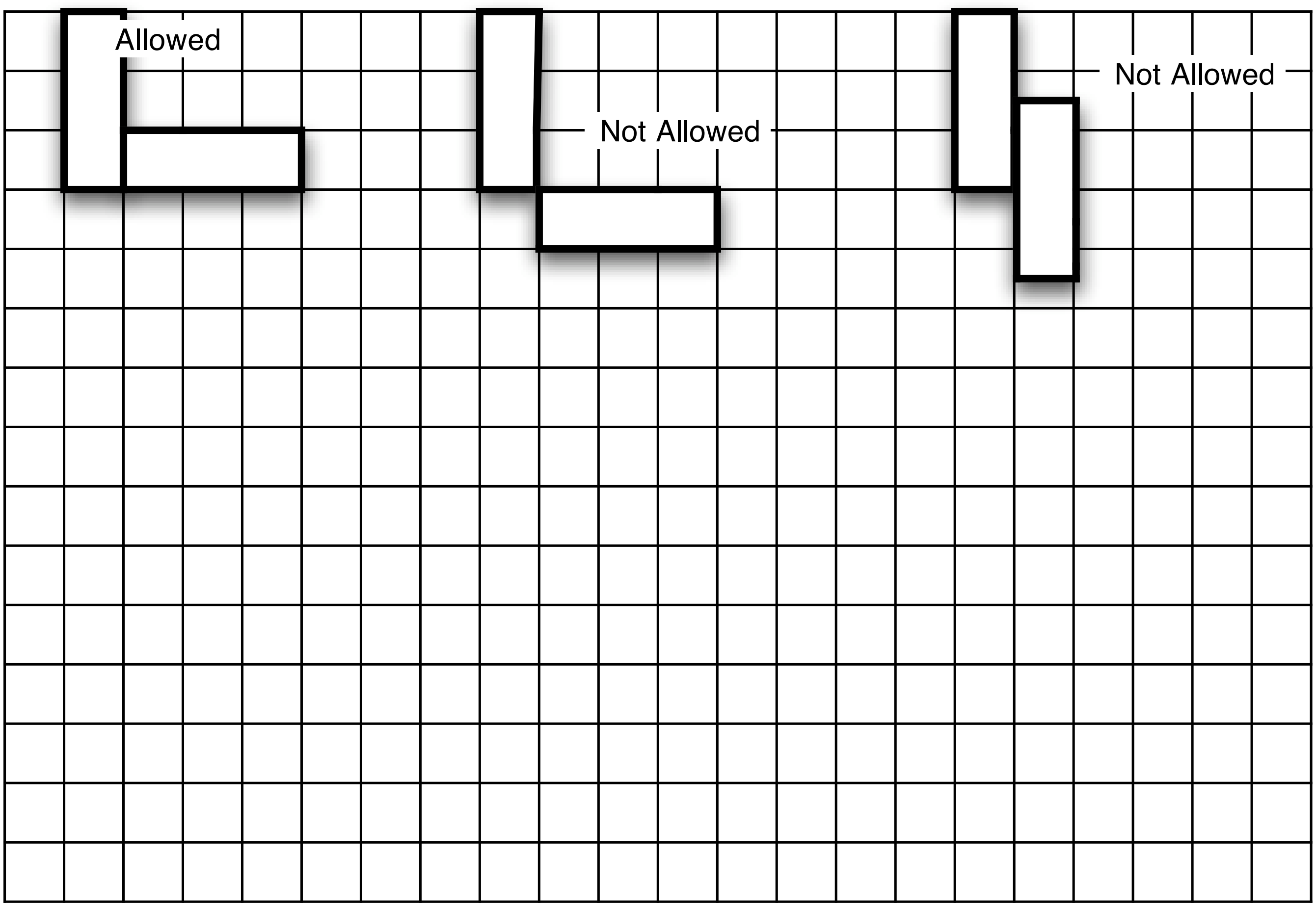
(4) [Section 2.2, Sets and logic]. A total of 50 people respond to a survey about whether they watch the two TV shows South Park and West Wing. 25 people in set A say that they watch South Park, regardless of whether they also watch West Wing. 30 people in set B say they watch West Wing, regardless of whether they also watch South Park. 10 say they watch neither. How many watch both? Explain your solution using both words (complete sentences!) and a Venn Diagram. Label anything you put in the Venn diagram.

Solution: ______ Explanation:

(5) [Section 1.4]. Five Starbucks are located in an equilateral triangular area, 2 miles on a side. Explain why a pair of them must be 1 mile or less from each other?



(6) [Section 1.2 – Problem 20]. You are given two 1 by 3 rectangles (called trominoes), as shown below. If the trominoes are placed edge to edge so that any pair that touch do so by exactly one, two, or three units (as in the example on the left), then which perimeters are possible for the shapes composed of two trominoes? Note that corner to corner placement shown in the middle is not allowed, and edges that meet along less than one unit as on the right, are also not allowed. Illustrate each perimeter that you find.
Possible perimeters: ____________________________

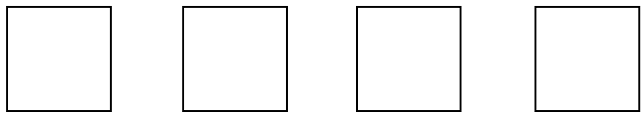


(7) [Chapter 1]. How many ways are there to make change for 20 cents, using dimes, nickels, and pennies? Give a detailed answer:

(8) [Section 1.4]. We saw how the pigeonhole principle is used in surprising ways. Use it to answer the following questions:

(a) If 250 students are to go on a field trip in 7 buses, then the pigeonhole principle guarantees that at least one bus must have at least _______ students on it (what is the maximum number guaranteed?)

(b) If five numbers are chosen from the set {1,2,3,4,6,8,12,24}, then there must be a pair of those numbers that have a product of exactly 24. Use the pigeonhole principle to explain why. You might want to use these boxes as pigeonholes, properly labeled:



Explanation:

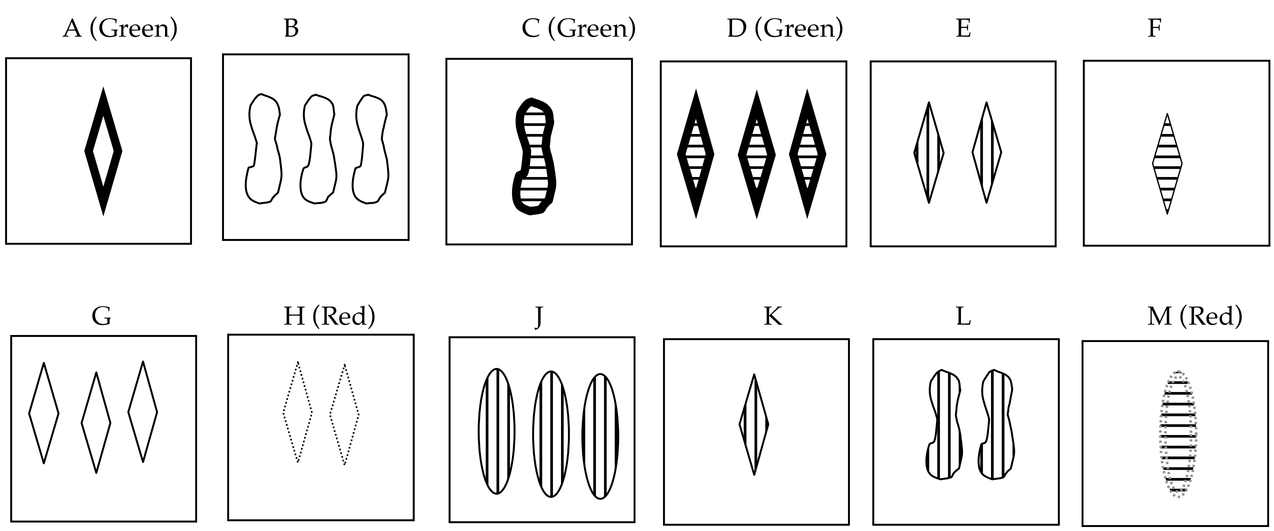
(9) [Section 2.4, 2.4, 2.1,1.3]. Circle the correct answer.
(a) True or False. Multiples of 5 are closed under multiplication.
(b) True or False? Multiples of 5 are closed under division.
(c) True or False? The complement of the complement of a set is the set itself.
(d) True or False? The third row of a triangle of numbers named for a French mathematician (but known in China centuries earlier) consists of 1,5,10,10,5, and 1.

(10) [Section 2.4]. Give a visual illustration for $2\ (1 + 2 + 4) = (2)(1) + (2)(2) + (2)(4)$

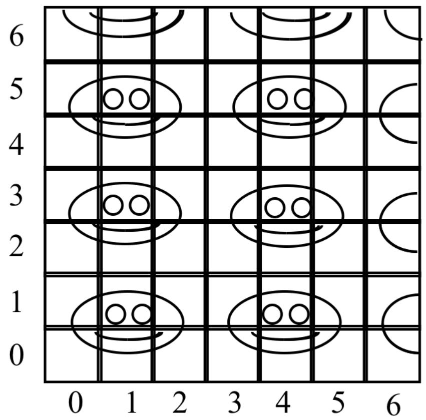
(11) [Game of Set - class activity – we'll do this on Tuesday. You can find out about this at the site http://www.setgame.com/set/index.html]. In the game of set, you must examine twelve cards, each of which has one of three shapes, one of three shadings, one of three colors, and one of three numbers. You must find three cards such that for each characteristic, all 3 cards are the same or are all different. For example, A, H, and G are three different "colors", three different numbers, all the same shape, and all are shaded white.

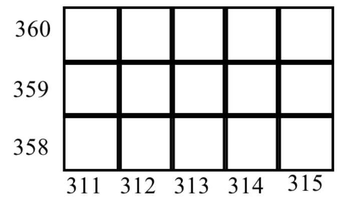
Find two other sets of three: ________ and ________

Note: Assume that A,C, and D are green; H and M are red; and the rest are purple.



(12) Pattern problem.
If this pattern continues throughout the plane, what pictures fall in the boxes shown on the right? (Draw the pattern.)





(13) [Section 1.3 - Patterns with the Fibonacci numbers 1,1,2,3,5,8, etc.] Give the next row of the following pattern, and complete the general formulafor the "odd case":

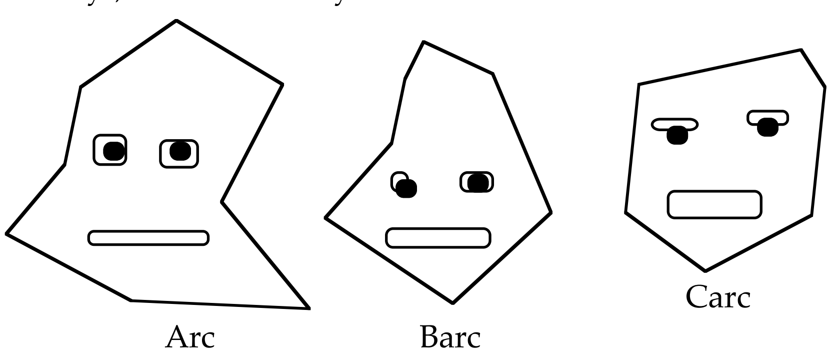
$(1)(3) = (1)(2) + 1$	$(F_1)\ (F_4) = (F_2)\ (F_3) + 1$
$(1)(5) = (2)(3) - 1$	$(F_2)\ (F_5) = (F_3)\ (F_4) - 1$
$(2)(8) = (3)(5) + 1$	$(F_3)\ (F_6) = (F_4)\ (F_5) + 1$
$(3)(13) = (5)(8) - 1$	$(F_4)\ (F_7) = (F_5)\ (F_6) - 1$

______________	________________________
(Show numbers here)	(Use the F notation here.)

General pattern or formula when k is an odd number: $(F_k)\ (F_{k+3}) =$ ________________

(14) [Logic problem]
Three space aliens Arc, Barc, and Carc stand before you. One of them has stolen all the Valentine cards in the entire world.

> Arc says, "Carc took the cards!"
> Barc says, "Neither I nor Carc did it."
> Carc says, "I don't have any Valentine cards."



Fortunately you know that at most one of them is lying, and so you can figure out who has taken the cards. Who is it? Explain in detail how you know. If you use a table, also include a verbal explanation.

(15) [Similar to the horse problem, which we will do in class soon!]. Logic.
On Monday Alex sold a printer to Bill for $20 and a modem to Dany for $20.
On Tuesday Bill sold the printer to Cora for $30, and Dany sold the modem to Cora also for $30.
On Wednesday Cora sold the printer to Dany for $40 and the modem to Bill for $40.
On Thursday Dany sold the printer to Alex for #50 and Bill sold the modem to Alex for $50.
At the end of the week who had lost or made how much from these sales? Circle lost or made and fill in the blanks:

Alex lost / made ________

Bill lost / made ________

Cora lost / made ________

Dany lost / made ________

QUESTIONS FOR LATER, WHEN WE'VE GOTTEN TO THESE SECTIONS:

(16) [Section 3.1]. Write the number 221 in

(a) Babylonian notation

(b) Mayan notation

(c) Egyptian notation

(d) Roman numerals

(e) Chinese "rod" system (Problem 10 in section 3.1)

(17) [Section 3.2] (a) Change to base ten: 10010110_2 = $\_\_\_\_\_\_\_\_\_\__{10}$

(b) Change to base two: 110_{10} = $\_\_\_\_\_\_\_\_\_\_\_\__2$