**Building Musical Scales** **Name:** ________________

(a) Let x take on each of the values 1,2,3,4 and 5, and then do each division $x \div (\frac{3}{2})$ and each multiplication $x \cdot (\frac{3}{2})$. Write down the results of these five divisions and five multiplications in the blanks below, after the original five numbers (write the results as fractions in lowest terms, not decimals):

1, 2, 3, 4, 5, _____, _____, _____, _____, _____, _____, _____, _____, _____, _____

(b) Place these fifteen numbers in ascending order (from lowest to highest) and cross off any duplicates.

____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

(c) If any of the numbers is less than 1, then multiply it by 2 repeatedly, until the result is between 1 and 2 (including 1 and2 themselves). If any of the numbers is greater than 2, then divide it by 2 repeatedly, until the result is a number between 1 and 2. Write the result as an "improper fraction." For example, if your number were 10, then

10 divided by 2 is 5
5 divided by 2 is $2\frac{1}{2}$
$2\frac{1}{2}$ divided by 2 is $1\frac{1}{4}$ or $\frac{5}{4}$, so write down $\frac{5}{4}$ instead of 10.
Again, cross off any duplicates:

____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

(You should have 8 different values.)

(d) Multiply each fraction by 264, the frequency, or number of vibrations per second, of middle C on the piano. The values other than middle C are close to the frequencies of the other white keys after middle C:

| C | D | E | F | G | A | B | C |
|---|---|---|---|---|---|---|---|
| ____ | ____ | ____ | ____ | ____ | ____ | ____ | ____ |

(e) These are a reasonable number of values for a musical scale. (The human hand can span eight notes on the piano, and the ratios of the frequencies of the notes are all fractions with fairly small numerators and denominators, which makes them sound pleasant together.)

Now use a graphing or scientific calculate all the values of 2 to the power $\frac{x}{12}$, for x equal to 0,1,2,3,4,5,6,7,8,9,10,11,12.. (When multiplied by 264, these will correspond to the twelve white and black keys on the piano that make up one "scale".)

| X (0 to 12) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $2^{x/12} =$ | | | | | | | | | | | | | |
| (c) Values | | | | | | | | | | | | | |

Which of these powers of 2 are close to the eight values you found above in part (c)? Write the values from part (c) below the value of 2 raised to a power that it is closest to. This demonstrates how the standard "equitempered" scale, which is derived from powers of 2, corresponds to the more ancient Pythagorean scale. The equitempered scale was championed by J.S. Bach and others, and is now used on most pianos. Musicians who play instruments which are not "pre-tuned" to a certain scale (such as violins or an opera singers voice) sometimes use the Pythagorean notes (or other simple fractions) instead of the equitempered notes, in order to create a more pleasing combination of notes.

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