

## Number Rules and Word Rules Too

Language does not always obey the same rules as numbers. The commutative, associative, and distributive properties for numbers sometimes do and sometimes do not work with words. In answering the questions in this handout, you may give examples from languages other than English. The handling of positives and negatives may vary from one language to another, and may reflect cultural and linguistic styles.

### Commutativity and Order.

The commutative property says that the order of the numbers to be added or multiplied does not matter:

$$3 + 4 = 4 + 3 = 7$$

$$3 \cdot 4 = 4 \cdot 3 = 12$$

This may not work for all arithmetic operations, for example,

$3 - 4 \neq 4 - 3$  (but they are the negatives of each other)

$3/4 \neq 4/3$  (but they are the reciprocals of each other)

These latter operations are sometimes said to be “anticommutative.”

The commutative property may or may not fail for words:

The big red apple  $\neq$  the red big apple

The ugly mean dog = the mean ugly dog

Problem 1. Give one example where the commutative property works for a word phrase, and one where it does not (and say which is which!):

## Distributivity.

The distributive property says that multiplication and division both distribute over addition and subtraction:

$$4(3 + 2) = 4 \cdot 3 + 4 \cdot 2 = 20$$

$$4(3 - 2) = 4 \cdot 3 - 4 \cdot 2 = 4$$

Problem 2. Show by means of a picture and an example why multiplication distributes over addition.

Also

$$(3 + 2) \cdot 4 = 3 \cdot 4 + 2 \cdot 4 =$$

$$(3 - 2) \cdot 4 = 3 \cdot 4 - 2 \cdot 4 =$$

Exponentiation and taking a root both distribute over multiplication and division:

$$(2 \cdot 3)^4 = 2^4 \cdot 3^4 = 1296$$

$$\sqrt{2 \cdot 3} = \sqrt{2} \cdot \sqrt{3} = 1.56508\dots$$

Problem 3. Give a different example to show that exponentiation and taking of roots distribute over division.

Problem 4. Give examples to show whether exponentiation and taking of roots distribute over addition and subtraction (and clearly say whether you think they do or do not!):

Here is an example due to Tom Lehrer of UC Santa Cruz (and famous for his quirky folk songs played on the Dr. Demento radio show) that shows the failure of the distributive property in language:

From a college housing form: “For roommate selection please describe your preference for age, major, smoking, and sex.”

Here is an example showing a successful distribution of negation:

“There is no smoking, drinking, or loud talking allowed in the library.”

Problem 5. Give examples of successful and unsuccessful use of distributivity of words (and say which is which!):

### **Positive versus Negative.**

Here are some examples of combinations of positives with negatives.

Positive times negative = negative times positive = negative:

“I confirmed I had never eaten sprouts.”

“I denied I had ever eaten sprouts.” = “I have not eaten sprouts.”

Double negative = positive:

“I will not neglect to call you” = “I will call you.”

Double negative = negative:

“I ain’t never gonna call you” = “I will not call you.”

Spanish: “No hay nadie que pueda contestar a esta pregunta.”

Literally this means: “There’s not no-one ie there is someone who can answer this question.”

But really it means: “There isn’t anyone who can can answer this question.”

An apocryphal tale about a philosophers’ convention relates the story of a philosopher who in a lecture about double negatives asserted that double negatives usually meant something positive, while double positives never meant anything negative. A voice from the back of the hall boomed out sarcastically, “Yeah, yeah.”

Or there is the saying, “Two wrongs don’t make a right, but three lefts do!”

Problem 6. Give some other examples of combinations of positives and negatives in language:

## Repetition.

“A rose is a rose is a rose is a rose.” – Gertrude Stein.

$$1 \cdot 1 \cdot 1 = 1$$

$$0 + 0 + 0 = 0$$

$$1 + 1 + 1 \neq 1$$

## Inverses.

$$x + (-x) = 0$$

Is “I had a dollar and lost a dollar” the same as “I never had a dollar”?

Is “I never felt better” the same as “I have felt worse”?

## Comparisons.

$x > y$  and  $y > 0$  together imply that  $x > 0$ .

“A dog is a better pet than a bird,” and “A bird is a better pet than no pet,” but “No pet is better than a dog.” Perhaps that is because this last sentence really means that “It is not the case that there exists a pet better than a dog.”

Problem 7. Give some examples of repetition, inverses, and comparison in language, and explain how they are either like or unlike the corresponding properties of numbers: