Math 44: Patterns and Modular (Clock) Arithmetic
Name: **SOLUTIONS**

IE!	AB	ER	CRO	MB	IE!	AB	ER	CRO	MB	IE!	AB	ER	CRO	MB	IE!
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

(1) The word "ABERCROMBIE!" begins in **cells 1, 6, 11, 16, 21, AND 26.** (Fill in the blanks with the next four starting cells.)

(2) Describe, in words, using the term "multiple," all the cells in which the word "ABERCROMBIE!" begins, assuming that the pattern continues indefinitely out to the right:
The "A"s occur in cells which are 1 more than a multiple of 5. It is not enough to say "in the multiples of 5," since the A's do not fall in the cells 0, 5, 10, 15, etc., which are actually multiples of 5. It is also not correct to say "they occur every 5," since, for example, the cells 2, 7, 12, 17, etc. also occur every 5, and this answer does not distinguish between the two sequences. It is also correct to say that the A's occur in cells which are congruent to 1, mod 5, since that means they are in cells which are 1 plus a multiple of 5.

(3) Where does the word "ABERCROMBIE!" fall in this set of cells, if we assume that the pattern continues to the left also? Draw it.

CRO	**MB**	**IE**	**AB**	**ER**	**CRO**	**MB**	**IE**	**AB**	**ER**	**CRO**	**MB**	**IE**
–22	**–21**	**–20**	**–19**	**–18**	**–17**	**–16**	**–15**	**–14**	**–13**	**–12**	**–11**	**–10**

The A's are in cells that are multiples of 5 plus 1: –19, –14, etc.

(4) Describe, in words, using the term "multiple," in which cells the letters "RO" fall, if we asssume that the pattern continues indefinitely to the left and to the right:

The RO's fall in cells that are 3 more than a multiple of 5. Or that are congruent to 3, mod 5. It is not correct to just say "every 5" or "they are multiples of 5." "Every 5" does not distinguish between other cell sequences that occur every 5, for example 1,6,11,16, etc. "Multiples of 5" is not correct because the numbers 3, 8, 13, 18, etc. are NOT multiples of 5!!

(5) Make up your own "linear" pattern, invent a puzzle question about it, and answer the question yourself:

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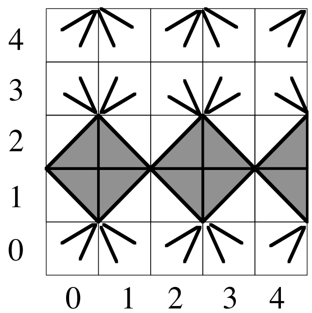
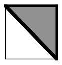
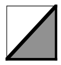
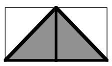


(6) The cells  and  fall in columns **2, 4, 6, 8, 10, and 12.** (Fill in the blanks with the numbers of the columns immediately preceeding and following columns 2, 4, and 6.)

(7) The cells  fall in rows **–2, 2, 6, 10, and 14**. (Answer as in (6).)

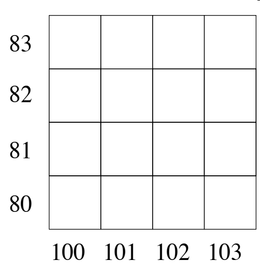
(8) Describe your answer to (6) using the terminology of modular or clock arithmetic, and also using the word "multiples," assuming the pattern continues throughout the plane:

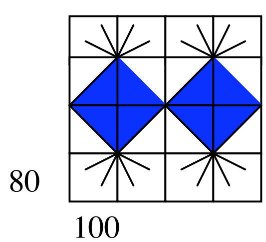
The columns are congruent to 0, mod 2. Or they are ≡ 0 (mod 2). They are also even numbers.

Describe your answer to (7) using the terminology of modular or clock arithmetic, and also using the word "multiples,"assuming the pattern continues throughout the plane:

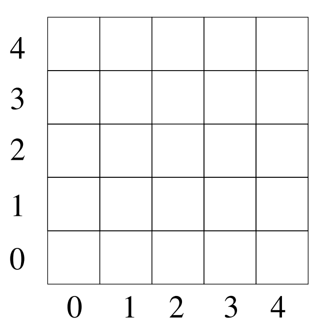
These columns are congruent to 2, mod 4, or they are ≡ 2 (mod 4). They are also 2 more than multiples of 4.

(9) Fill in with the pattern of problems 6-8:





(10) create your own pattern:



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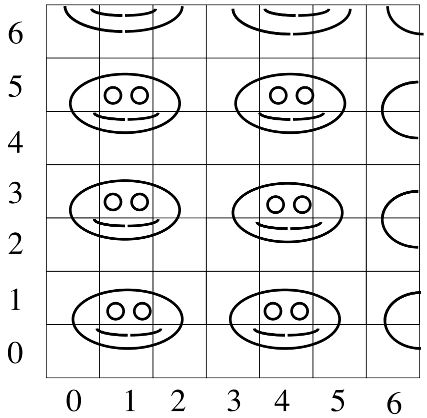
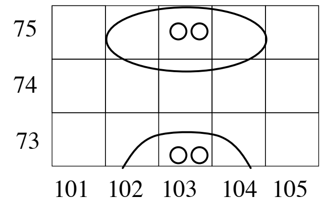
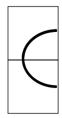




The eyes are in rows 73 and 75, and column 103

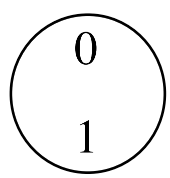
(11) The left hand side of the face  falls in columns 0, 3, 6, **9, 12, 15, and 18.** (Fill in with the next columns after 0, 3, and 6.)

(12) The eyes fall in rows 1, 3, 5, **7, 9, 11, and 13.** (Fill in with the rows following 1, 3, and 5.)

(13) What kind of numbers are the eye-rows? (Use the terminology of modular arithmetic.)
Odd numbers, or numbers congruent to 1, mod 2. Also: numbers 1 more than a multiple of 2.

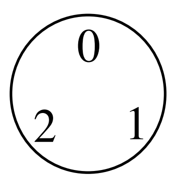
(14) What kind of numbers are the eye-columns? Use the terminology of modular arithmetic to describe them; assume the pattern continues throughout the plane.
1 more than a multiple of 3, or congruent to 1, mod 3.

(15) Fill in with either 0 or 1:
$73 \equiv 1 \pmod 2$
$74 \equiv 0 \pmod 2$
$75 \equiv 1 \pmod 2$



Clock with 2 numbers

Fill in with either 0, 1, or 2:
$101 \equiv 2 \pmod 3$
$102 \equiv 0 \pmod 3$
$103 \equiv 1 \pmod 3$
$104 \equiv 2 \pmod 3$
$105 \equiv 0 \pmod 3$



Clock with 3 numbers

(16) Fill in the chart on the right at the top of the page with the pattern of faces.