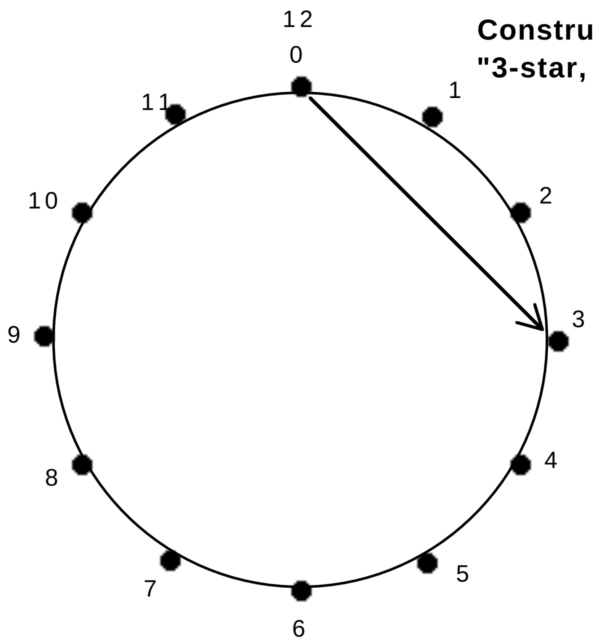
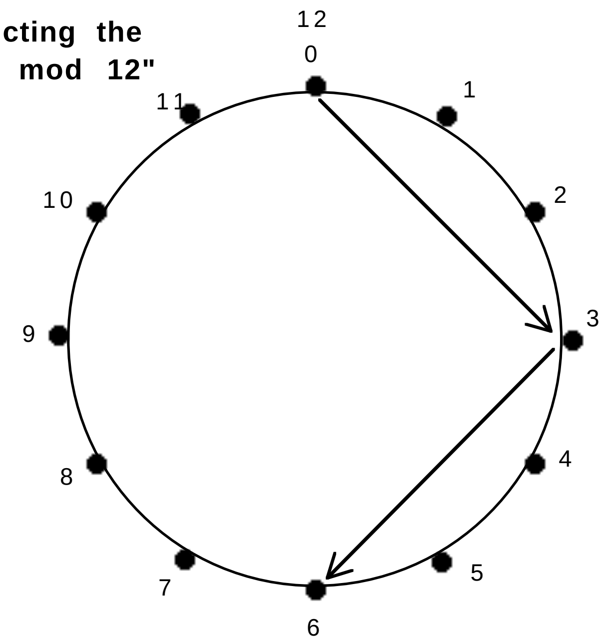
Poinsot Stars **Math 44** **Name: __________**

Named after a French mathematician who investigated them in the 1800s.
("==" is used in the graphic below to mean "congruent to".)

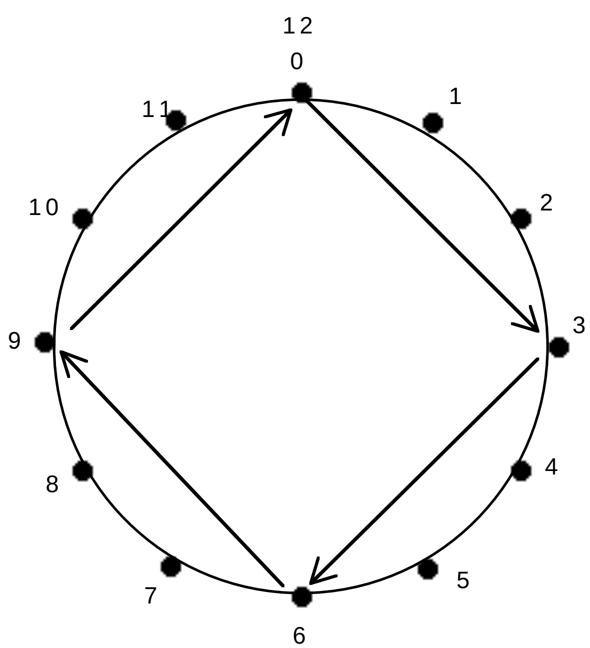
Constructing the "3-star, mod 12"



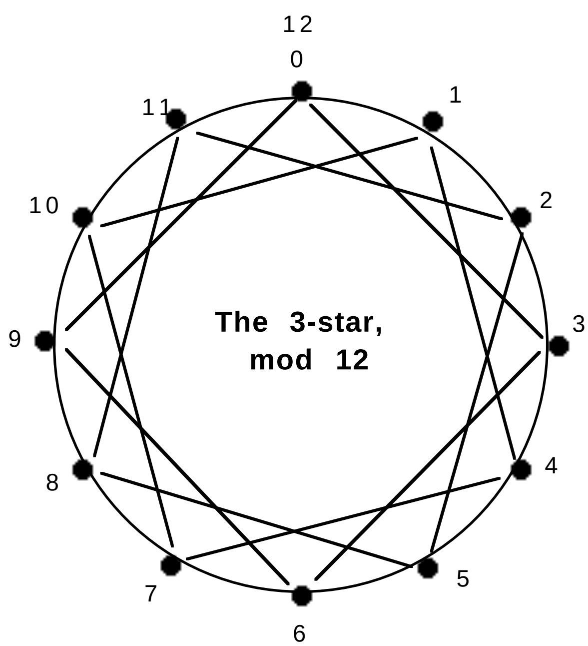
First we draw an arrow from 0 to 3 to indicate that we have added 3 once.



Next we draw an arrow from 3 to 6 to show that we have added one more 3: 6==3+3



Continue in this way until the arrow s come back to where they started:
12==0==3+3+3+3==4(3) (mod 12)
Notice that this first part of the star is just a square!

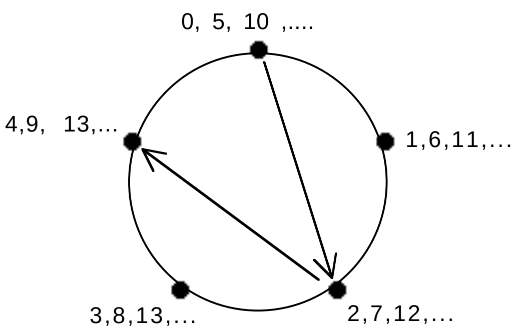


Now draw the number stars (squares!) starting at 1 and at 2. Together these three squares make up the "3-star, mod 12".

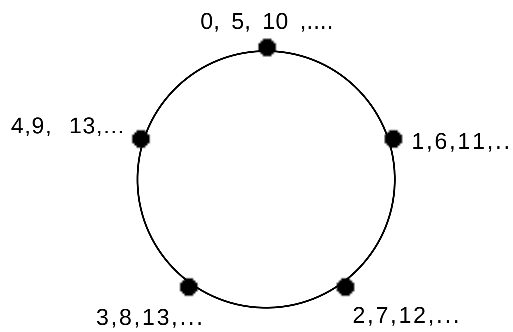
(1) After how many 3 hour periods, beginning at 2 o'clock, will the clock show 9 o'clock? How can you use the charts above to help answer the question? (If it's not possible, then explain why not!)

(2) How is question (1) like solving the "congruence equation":
$2 + 3x \equiv 9$?

(3) Complete the following charts:

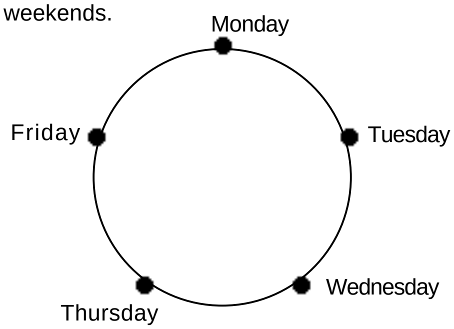


Complete this circular table of the multiples of 2, mod 5, starting at 0.



Draw the arrows for the multiples of 3, mod 5, starting at 0.

You might think of what is going on in terms of a five -day week at an office building. Suppose a cleaning crew cleans the building every third weekday. Use the circles to show the sequence of days on which the building will be cleaned, assuming they start on Monday, and do not work weekends.

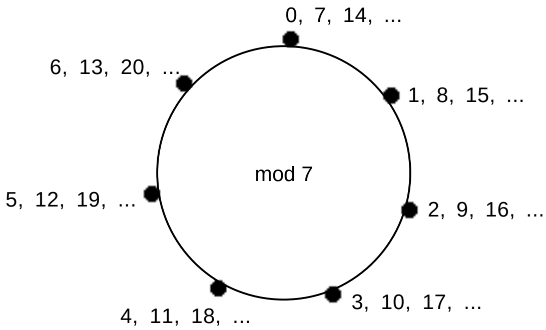


(4) Sometimes we say that $-2 \equiv 3 \pmod 5$. Explain how this would appear visually. How does this make the multiples of -2 and 3 look alike?

(5) We might ask the question, "After how many two-day periods will the work crew clean a building on a Thursday, assuming they start on a Wednesday?" How can you use the charts above to find the answer?

Why is this question like solving $2x + 2 \equiv 3 \pmod 5$?

Another way to see that $-2 \equiv 3 \pmod 5$, is to notice that division of 3 by 5 leaves the remainder 3, and so does division of -2 by 5:
$3 \equiv 5(0) + 3 \pmod 5$
$-2 \equiv 5(-2) + 3 \pmod 5$
(We have turned around the divisions and made them look like multiplications!)

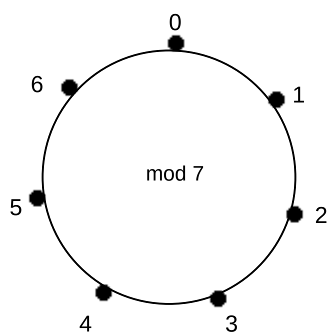


(6) Make the arrows for the multiples of 3, mod 7, starting at 0.

Solve the "congruence"
3x == 8 (mod 7),
by following the arrows around the diagram, to see how many multiples of 3 you need to get to 8 from 0..

The solution is x == ___ (mod 7)

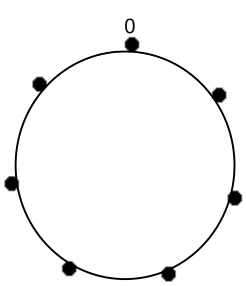
You might think of this problem, 3x == 8 (mod 7), in the following terms: beginning on Sunday (day 0), suppose that you go to the gym every third day. After how many three day periods will you find yourself in the gym on a Monday?



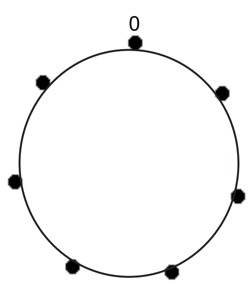
(7) Make the arrows for the multiples of 3 (mod 7), beginning at the 2. Then use them to solve this congruence:

3x + 2 == 10 (mod 7)

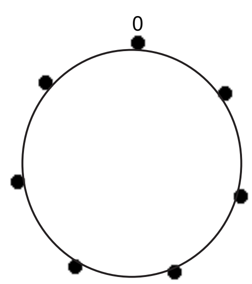
Make up a "story problem" to go with this congruence:



(8) Make the 2-star, starting at 0.



Make the 1-star, starting at 0.



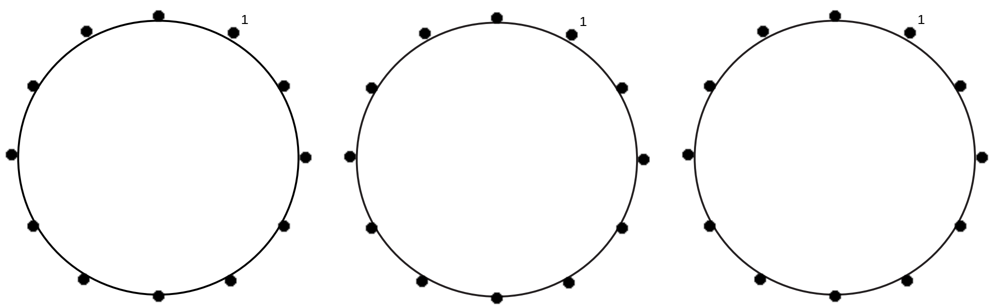
Make the 6-star, starting at 0.

(9) A runner does a long-distance workout every fourth day, including weekends. Show how she can use the Poinsot star diagram to map out her workouts.

(10) September 1st and December 1st both fall on Saturday in 2001. For that matter, Sep. x and Dec. x will fall on the same day of the week in 2001. Explain. (Hint: use the ideas of mod 7).

(11) Find two other months in which the days of the months having the same numbers (for example, the 18th of each month), fall on the same days of the week.

(12) Show the 2-stars, the 5-star, and the 8-stars on the 12-clock (if a star does not "go" to each point, then start it again at an untouched point, until all the points are included in some star):



(13) Explain why 8 hour shifts "work" with our base-12 or mod 12 (or mod 24) clock, whereas 5 hour shifts get "messy":

(14) Why, if you start at star on one of these clocks, must you eventually return to your starting place?

(15) On the 12-clock, what is the relationship between the n-star(s) and the factors that n might have in common with the number 12?

(16) Investigate stars on the 15-clock. What patterns can you find ? Which multiples lead to stars that eventually go to every point? What connections do you see to the 5-clock in earlier exercises?