

SCORE: ____ / 10 POINTS

NO CALCULATORS ALLOWED

Evaluate the following limits algebraically. Show all algebraic work.

[a] $\lim_{x \rightarrow 3} \frac{\frac{1}{x^2} - \frac{1}{9}}{\frac{1}{x} - \frac{1}{3}} = \lim_{x \rightarrow 3} \frac{\frac{1}{x^2} - \frac{1}{9}}{\frac{1}{x} - \frac{1}{3}} \cdot \frac{9x^2}{9x^2}$ SCORE: ____ / 3 POINTS

$$= \lim_{x \rightarrow 3} \frac{9 - x^2}{9x - 3x^2}$$

$$= \lim_{x \rightarrow 3} \frac{(3-x)(3+x)}{3x(3-x)} = \lim_{x \rightarrow 3} \frac{3+x}{3x} = \frac{3+3}{3(3)} = \frac{2}{3}$$

[b] $\lim_{x \rightarrow 5} \frac{3x-15}{\sqrt{x-1}-2} = \lim_{x \rightarrow 5} \frac{(3x-15)(\sqrt{x-1}+2)}{x-1-4}$ SCORE: ____ / 3 POINTS

$$= \lim_{x \rightarrow 5} \frac{3(x-5)(\sqrt{x-1}+2)}{x-5}$$

$$= \lim_{x \rightarrow 5} 3(\sqrt{x-1}+2) = 3(\sqrt{5-1}+2) = 12$$

[c] $\lim_{x \rightarrow -1} \frac{x^3+1}{x^2-1} = \lim_{x \rightarrow -1} \frac{(x+1)(x^2-x+1)}{(x+1)(x-1)}$ SCORE: ____ / 3 POINTS

$$= \lim_{x \rightarrow -1} \frac{x^2-x+1}{x-1}$$

$$= \frac{(-1)^2 - (-1) + 1}{-1 - 1} = -\frac{3}{2}$$

Give an example of functions f and g such that $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$ exists, but $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 0} g(x)$ do not exist. SCORE: ____ / 1 POINT

$$f(x) = \frac{1}{x}$$

$$g(x) = \frac{1}{x^2}$$

$$\frac{f(x)}{g(x)} = x$$

INFINITELY MANY
POSSIBILITIES