

SCORE: ____ / 10 POINTS

NO CALCULATORS ALLOWED

Prove that $\frac{d}{dx} \cos x = -\sin x$ from the definition of the derivative.

SCORE: ____ / 3 POINTS

You may use the two limits proved in class without reproving them.

$$\begin{aligned} \frac{d}{dx} \cos x &= \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} \quad \frac{1}{2} \\ &= \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \quad \frac{1}{2} \\ &= \lim_{h \rightarrow 0} \left(\cos x \frac{\cos h - 1}{h} - \sin x \frac{\sin h}{h} \right) \quad 1 \\ &= 0 \cdot \cos x - 1 \cdot \sin x = -\sin x \end{aligned}$$

Find the derivatives of the function $f(x) = 5^{\tan 4x}$. SIMPLIFY YOUR ANSWER.

SCORE: ____ / 2 POINTS

$$\begin{aligned} f'(x) &= 5^{\tan 4x} (\ln 5) (\sec^2 4x) (4) \\ &= (4 \ln 5) 5^{\tan 4x} \sec^2 4x \quad \frac{1}{2} \text{ POINT EACH} \end{aligned}$$

If $g(x)$ is the inverse of $f(x) = x^3 + x - 3$, find $g'(7)$.

SCORE: ____ / 2 POINTS

$$\begin{aligned} \frac{1}{2} \quad f(2) &= 2^3 + 2 - 3 = 7 \\ \text{so } g(7) &= 2 \\ \text{so } g'(7) &= \frac{1}{f'(2)} = \frac{1}{3(2)^2 + 1} = \frac{1}{13} \quad \frac{1}{2} \end{aligned}$$

If $f(x) = \cos x$, find $f^{(75)}(x)$. You must explain why your answer is correct.

SCORE: ____ / 3 POINTS

$$\begin{aligned} f(x) &= \cos x & f^{(4)}(x) &= \cos x & f^{(8)}(x) &= \cos x \\ f'(x) &= -\sin x & f^{(5)}(x) &= -\sin x & & \text{ETC.} \\ f''(x) &= -\cos x & f^{(6)}(x) &= -\cos x \\ f'''(x) &= \sin x & f^{(7)}(x) &= \sin x \end{aligned}$$

CYCLE OF DERIVATIVES REPEATS EVERY 4TH TIME

$$f^{(72)}(x) = f(x)$$

$$f^{(75)}(x) = f'''(x) = \sin x$$