

SCORE: ____ / 10 POINTS

NO CALCULATORS ALLOWED

Knock knock ! Is anyone out there ? I'm trapped inside a black bar and no one seems to notice I'm here.

Find the linear approximation of $f(x) = \tan x$ at $x = \frac{\pi}{4}$. Use the approximation to estimate $f\left(\frac{3}{4}\right)$.

SCORE: ____ / 4 POINTS

$$\begin{aligned} L(x) &= f\left(\frac{\pi}{4}\right) + f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right) \\ &= \tan \frac{\pi}{4} + (\sec^2 \frac{\pi}{4})\left(x - \frac{\pi}{4}\right) \\ &= 1 + 2\left(x - \frac{\pi}{4}\right) \\ f\left(\frac{3}{4}\right) &\approx L\left(\frac{3}{4}\right) = 1 + 2\left(\frac{3}{4} - \frac{\pi}{4}\right) \\ &= 1 + \frac{3}{2} - \frac{\pi}{2} \\ &= \frac{5-\pi}{2} \end{aligned}$$

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Use an appropriate linear approximation to estimate $\sqrt[4]{79}$.

SCORE: ____ / 6 POINTS

$$\begin{aligned} f(x) &= x^{\frac{1}{4}} \\ x_0 &= 81 \\ L(x) &= f(x_0) + f'(x_0)(x - 81) \\ &= 81^{\frac{1}{4}} + \frac{1}{4}(81)^{-\frac{3}{4}}(x - 81) \\ &= 3 + \frac{1}{4} \frac{1}{27}(x - 81) \\ &= 3 + \frac{1}{108}(x - 81) \\ \sqrt[4]{79} &= f(79) \approx L(79) = 3 + \frac{1}{108}(79 - 81) \\ &= 3 - \frac{1}{54} \\ &= 2 \frac{53}{54} \text{ OR } \frac{161}{54} \end{aligned}$$