

SCORE: ____ / 10 POINTS

NO CALCULATORS ALLOWED

2.71828182845904523536028747135266249775724709369995957496696762772407663035354759457138217852516642742746639193200

Give the definition of the term "critical number". Your answer should be in a complete sentence.

SCORE: ____ / 1 POINT

$x=a$ IS A CRITICAL NUMBER OF f
IF a IS IN THE DOMAIN OF f AND $f'(a)=0$ OR $f'(a)$ IS UNDEFINED.

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Give the definition of the term "concave up". Your answer should be in a complete sentence.

SCORE: ____ / 1 POINT

f IS CONCAVE UP ON I IF f' IS INCREASING ON I

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Find all critical numbers of the following functions and, using the first derivative test, classify them as local maxima or local minima or neither. SHOW ALL WORK.

SCORE: ____ / 5 POINTS

[1] $f(x) = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$

$f'(x) = \frac{5}{3}x^{\frac{2}{3}} - \frac{10}{3}x^{-\frac{1}{3}}$

f' IS UNDEFINED IF $x=0$ (IN DOMAIN)

$f'=0$ IF $\frac{5}{3}x^{-\frac{1}{3}}(x-2)=0$ IE. $x=2$

$\frac{5}{3}x^{-\frac{1}{3}}$	-	+	+	+
$x-2$	-	-	+	+
f'	+	0 MAX $\frac{1}{2}$	-	2 MIN $\frac{1}{2}$

[2] $f(x) = 6x^4 - 8x^3 + 3x^2 - 1$

$f'(x) = 24x^3 - 24x^2 + 6x$

$= 6x(4x^2 - 4x + 1)$

$= 6x(2x-1)^2 = 0$

IF $x=0$ OR $\frac{1}{2}$

$6x$	-	+	+	+
$(2x-1)^2$	+	+	+	+
f'	-	0 MIN $\frac{1}{2}$	+	NEITHER $\frac{1}{2}$

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Find all inflection points of the function $f(x) = xe^{-x}$. SHOW ALL WORK.

SCORE: ____ / 3 POINTS

$f'(x) = e^{-x} - xe^{-x}$

$f''(x) = -e^{-x} - e^{-x} + xe^{-x} = e^{-x}(-2+x) = 0$

IF $x=2$

e^{-x}	+	+	
$-2+x$	-	+	
f''	-	2 INFLECTION POINT	+

$\frac{1}{2}$ POINT EACH

NOTE: YOU MUST DO THE SIGN ANALYSIS ON f'' TO KNOW $x=2$ IS AN INFLECTION POINT

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3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328

Give the definition of the term "global maximum". Your answer should be in a complete sentence.

SCORE: ____ / 1 POINT

f HAS A GLOBAL MAXIMUM AT $x=a$ IN THE SET S
IF $f(x) \leq f(a)$ FOR ALL $x \in S$

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328

Give the definition of the term "decreasing". Your answer should be in a complete sentence.

SCORE: ____ / 1 POINT

f IS DECREASING ON I IF WHENEVER $x_1 < x_2$ AND $x_1, x_2 \in I$
WE HAVE $f(x_1) > f(x_2)$

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SCORE: ____ / 5 POINTS

[1] $f(x) = 6x^4 - 8x^3 + 3x^2 - 1$

[2] $f(x) = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$

SEE OTHER KEY

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328

Find all inflection points of the function $f(x) = xe^{-x}$. SHOW ALL WORK.

SCORE: ____ / 3 POINTS

SEE OTHER KEY