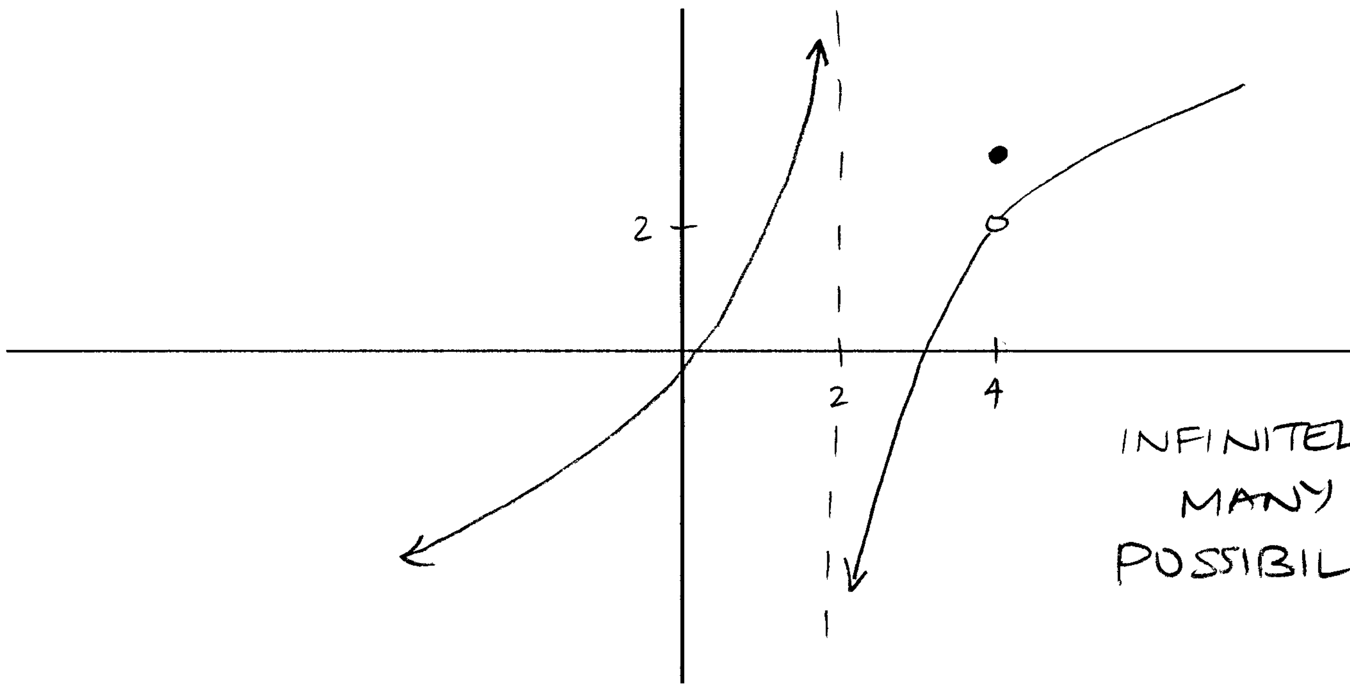
Math 1A (7:30am – 8:20am)
Midterm 1
Thu Oct 9, 2008

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

CALCULATOR ALLOWED ON THIS SECTION

Sketch a graph of a function f with **all** the following properties. **SCORE: ___ / 10 POINTS**

The domain of f is all real numbers except $x = 2$,
f has a removable discontinuity at $x = 4$ and a non-removable discontinuity at $x = 2$,
$\lim_{x \to 4+} f(x) = 2$, and $\lim_{x \to 2+} f(x) = -\infty$.



INFINITELY MANY POSSIBILITIES

A function f is continuous from the right at $x = a$ if $\lim_{x \to a+} f(x) = f(a)$. **SCORE: ___ / 8 POINTS**

If $f(x) = \begin{cases} cx^2 + 1 & if\ x < 2 \\ -1 & if\ x = 2 \\ cx + 4 & if\ x > 2 \end{cases}$, find all values of c so that f is continuous from the right at $x = 2$ (if possible).

Show all algebraic work.

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (cx + 4) = 2c + 4$$

$$f(2) = -1$$

$$2c + 4 = -1$$

$$c = \frac{-5}{2}$$

Find the one statement below which is false. Give a counterexample showing why it is false. **SCORE: ___ / 8 POINTS**

Statement 1: If $\lim_{x\to a} f(x) = +\infty$ and $\lim_{x\to a} g(x) = +\infty$, then $\lim_{x\to a} f(x)g(x) = +\infty$.

Statement 2: If $\lim_{x\to a} f(x) = +\infty$ and $\lim_{x\to a} g(x) = +\infty$, then $\lim_{x\to a}[f(x) - g(x)] = 0$.

Statement 3: If $\lim_{x\to a} f(x) = +\infty$ and $\lim_{x\to a} g(x) = +\infty$, then $\lim_{x\to a}[f(x) + g(x)] = +\infty$.

$$\lim_{x\to 0}\left(\frac{1}{x^2}+1\right) = +\infty \qquad \lim_{x\to 0}\frac{1}{x^2} = +\infty$$

$$\lim_{x\to 0}\left(\frac{1}{x^2}+1-\frac{1}{x^2}\right) = \lim_{x\to 0} 1 = 1$$

Let $f(x) = -1 + x\cos x$. **SCORE: ___ / 16 POINTS**

[a] Prove that $f(x)$ has a zero in the interval $[-8, 8]$. **<u>You must justify your argument properly as shown in class.</u>**

f IS CONT. ON $[-8,8]$ SINCE IT IS A SUM AND PRODUCT OF CONT. FUNCTIONS

$f(-8) > 0$

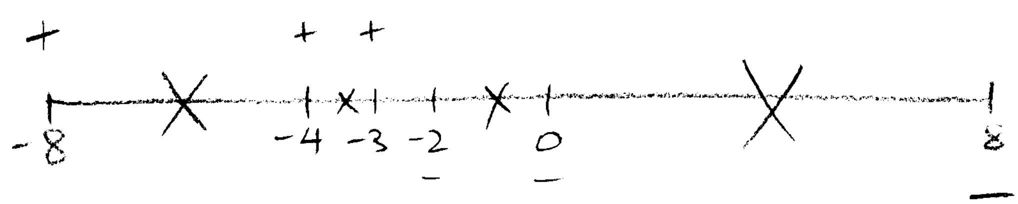
$f(8) < 0$ AND $f(8) \neq f(-8)$

$f(8) < 0 < f(-8)$

SO, BY IVT, THERE IS A $c \in (-8,8)$ SUCH THAT $f(c)=0$

IE $f(x)$ HAS A ZERO IN $[-8,8]$

[b] Use the method of bisections on the interval $[-8, 8]$ to find an interval of width 1 that contains a zero.
<u>You must show all x-values you used in the method of bisections.</u>



$f(0) < 0$

$f(-4) > 0$

$f(-2) < 0$

$f(-3) > 0$

$[-3, -2]$