Math 1A (7:30am – 8:20am)
Midterm 1
Thu Oct 9, 2008

SCORE: ___ / 120 POINTS

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

CALCULATOR NOT ALLOWED ON THIS SECTION

Determine the intervals on which $f(x)=\sqrt{x^2-4}$ is continuous. **Show all algebraic work.** **SCORE: ___ / 8 POINTS**

DOMAIN: $x^2-4 \geq 0$

$(x+2)(x-2) \geq 0$

		-2		2	
$x+2$	$-$		$+$		$+$
$x-2$	$-$		$-$		$+$
x^2-4	$+$	0	$-$	0	$+$

DOMAIN $=(-\infty,-2]\cup[2,\infty)$

$\lim_{x\to -2^-}\sqrt{x^2-4}=0=\sqrt{(-2)^2-4}$

$\lim_{x\to 2^+}\sqrt{x^2-4}=0=\sqrt{2^2-4}$

SO $f(x)$ IS CONT. ON $(-\infty,-2]$ AND $[2,\infty)$

Find all vertical asymptotes of $f(x)=\dfrac{x-1}{16-8x+x^2}$. For each vertical asymptote, determine whether $f(x)\to\infty$ or $f(x)\to-\infty$ on each side of the asymptote. **Show all relevant work.** **SCORE: ___ / 12 POINTS**

$f(x)=\dfrac{x-1}{(x-4)^2}$

$(x-4)^2=0$

$x=4$

$\lim_{x\to 4^-}\dfrac{x-1}{(x-4)^2}=+\infty$ $\quad \dfrac{3}{0^+}$

$\lim_{x\to 4^+}\dfrac{x-1}{(x-4)^2}=+\infty$ $\quad \dfrac{3}{0^+}$

State the Intermediate Value Theorem. **SCORE: ___ / 6 POINTS**

IF f IS CONT. ON $[a,b]$ AND $f(a) \neq f(b)$
AND W IS BETWEEN $f(a)$ AND $f(b)$
THEN THERE EXISTS A $c \in (a,b)$
SUCH THAT $f(c) = W$

Complete the precise definition of a limit: **SCORE: ___ / 8 POINTS**

$\lim_{x \to a} f(x) = L$ if FOR ALL $\varepsilon > 0$, THERE EXISTS A $\delta > 0$ SUCH THAT WHENEVER $0 < |x-a| < \delta$, WE HAVE $|f(x) - L| < \varepsilon$

Suppose that the size of the pupil (in millimeters) of a certain animal is given by $f(x) = \dfrac{48x^{-0.4} + 72}{3x^{-0.4} + 8}$, **SCORE: ___ / 8 POINTS**

where x is the intensity of the light on the pupil. Find the size of the pupil as the light fades to nothing. **<u>Show all algebraic work.</u>**

$$\lim_{x \to 0^+} \frac{48x^{-0.4} + 72}{3x^{-0.4} + 8} = \lim_{x \to 0^+} \frac{48x^{-0.4} + 72}{3x^{-0.4} + 8} \cdot \frac{x^{0.4}}{x^{0.4}}$$

$$\frac{\infty}{\infty}$$

$$= \lim_{x \to 0^+} \frac{48 + 72x^{0.4}}{3 + 8x^{0.4}}$$

$$= \frac{48 + 0}{3 + 0}$$

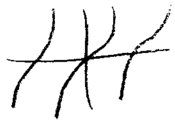
$$= 16 \text{ mm}$$

Evaluate the following limits. Your answer should be a number, $+\infty$, $-\infty$ or "DNE" (if the other answers do not apply. **Show all relevant work as demonstrated in class. If an answer is "DNE", explain briefly.** **SCORE: ___ / 36 POINTS**

$\lim\limits_{x\to\infty} \tan(\ln x)$ DNE

$\lim\limits_{x\to\infty} \ln x = +\infty$

$\lim\limits_{x\to+\infty} \tan x$ DNE



BECAUSE $\tan x \to \pm\infty$ AT $\frac{\pi}{2} + n\pi$ FOR ALL INTEGERS n

$\lim\limits_{x\to 3^-} f(x)$ where $f(x) = \begin{cases} 2x-7 & \text{if } x<1 \\ 5-3x & \text{if } 1<x<4 \\ x+2 & \text{if } x>4 \end{cases}$ $= -4$

$\lim\limits_{x\to 3^-} (5-3x) = -4$

$\lim\limits_{x\to -5} \dfrac{x+5}{\dfrac{6}{x+7}-3} = -\dfrac{2}{3}$

$\frac{0}{0}$

$\lim\limits_{x\to -5} \dfrac{x+5}{\frac{6}{x+7}-3} \cdot \dfrac{x+7}{x+7}$

$= \lim\limits_{x\to -5} \dfrac{(x+5)(x+7)}{6-3(x+7)}$

$= \lim\limits_{x\to -5} \dfrac{(x+5)(x+7)}{-3x-15}$

$= \lim\limits_{x\to -5} \dfrac{\cancel{(x+5)}(x+7)}{-3\cancel{(x+5)}} = -\dfrac{2}{3}$

$\lim\limits_{x\to 2} \dfrac{6-3x}{5-\sqrt{9x+7}} = \dfrac{10}{3}$

$\frac{0}{0}$

$\lim\limits_{x\to 2} \dfrac{6-3x}{5-\sqrt{9x+7}} \cdot \dfrac{5+\sqrt{9x+7}}{5+\sqrt{9x+7}}$

$= \lim\limits_{x\to 2} \dfrac{3(2-x)(5+\sqrt{9x+7})}{25-(9x+7)}$

$= \lim\limits_{x\to 2} \dfrac{3(2-x)(5+\sqrt{9x+7})}{18-9x}$

$= \lim\limits_{x\to 2} \dfrac{\cancel{3}\cancel{(2-x)}(5+\sqrt{9x+7})}{\cancel{9}_{3}\cancel{(2-x)}}$

$= \dfrac{10}{3}$

$\lim\limits_{x\to -\infty} \arctan(1-e^x) = \dfrac{\pi}{4}$

$\lim\limits_{x\to -\infty} (1-e^x) = 1-0 = 1$

$\lim\limits_{x\to 1} \arctan x = \dfrac{\pi}{4}$

$\lim\limits_{x\to 1} \dfrac{x^2-5x-4}{x^2-4x-3} = \dfrac{1-5-4}{1-4-3} = \dfrac{-8}{-6} = \dfrac{4}{3}$