Math 1A (7:30am – 8:20am)
Midterm 2
Tue Nov 4, 2008 **ELECTION DAY – DID YOU VOTE YET ?**

SCORE: ___ / 120 POINTS

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

THIS IS A NO-CALCULATOR MIDTERM

State the Mean Value Theorem. **SCORE: ___ / 6 POINTS**

IF f IS CONT. ON $[a,b]$, $1\frac{1}{2}$
AND f IS DIFF. ON (a,b), $1\frac{1}{2}$
THEN THERE IS SOME $c \in (a,b)$, 1
SUCH THAT $f'(c) = \frac{f(b)-f(a)}{b-a}$ 2

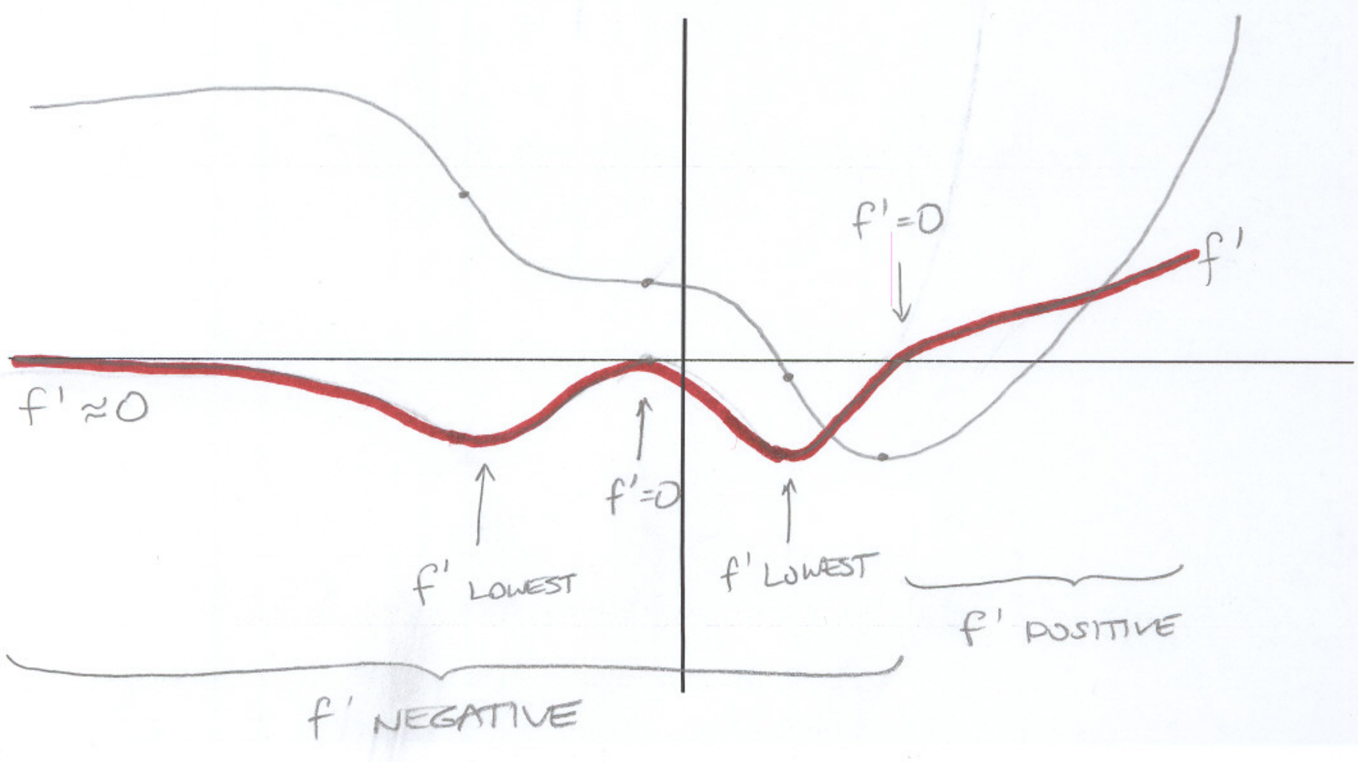
State the definition of the derivative of a function at a point. **SCORE: ___ / 4 POINTS**

THE DERIVATIVE OF f AT $x=a$ IS, 1

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$ 3

IF THE LIMIT EXISTS

The graph of $f(x)$ is shown below. Sketch a graph of $f'(x)$ on the same axes. **SCORE: ___ / 8 POINTS**



Evaluate the following derivatives. **Simplify anwers where convenient.** SCORE: ___ / 40 POINTS

[a] Find $f'(x)$ if $f(x) = \tan^{-1}\frac{1}{x^2}$.

8

$$f'(x) = \underbrace{\frac{1}{1+\left(\frac{1}{x^2}\right)^2}}_{3} \cdot \underbrace{\frac{-2}{x^3}}_{3}$$

$$= \underbrace{\frac{-2}{x^3+\frac{1}{x}}}_{1}$$

$$= \underbrace{\frac{-2x}{x^4+1}}_{1}$$

[b] Find $f'(x)$ if $f(x) = x^{\tan x}$.

10

$$\underbrace{\ln f(x) = \tan x \ln x}_{2}$$

$$\underbrace{\frac{1}{f(x)} f'(x)}_{2} = \underbrace{\sec^2 x \ln x}_{2} + \underbrace{\frac{\tan x}{x}}_{2}$$

$$f'(x) = \underbrace{x^{\tan x}\left(\sec^2 x \ln x + \frac{\tan x}{x}\right)}_{2}$$

$$= x^{\tan x - 1}(x \sec^2 x \ln x + \tan x)$$

[c] Find $g'(x)$ if $g(x) = \ln\left(x + (f(x))^2\right)$ where $f(x)$ is an unspecified function.

6

$$g'(x) = \underbrace{\frac{1}{(x+(f(x))^2)}}_{2} \cdot (\underbrace{1}_{1} + \underbrace{2f(x)f'(x)}_{2})$$

$$= \underbrace{\frac{1+2f(x)f'(x)}{(x+(f(x))^2)^2}}_{1}$$

[d] Find $\frac{d^2y}{dt^2}$ if $y = \frac{4t^3 - 5t^2 - 6}{\sqrt[3]{t}}$.

6

$$y = 4t^{\frac{8}{3}} - 5t^{\frac{5}{3}} - 6t^{-\frac{1}{3}}$$

$$\underbrace{\frac{dy}{dt} = \frac{32}{3}t^{\frac{5}{3}} - \frac{25}{3}t^{\frac{2}{3}} + 2t^{-\frac{4}{3}}}_{3}$$

$$\underbrace{\frac{d^2y}{dt^2} = \frac{160}{9}t^{\frac{2}{3}} - \frac{50}{9}t^{-\frac{1}{3}} - \frac{8}{3}t^{-\frac{7}{3}}}_{3}$$

[e] Find $\frac{dy}{dx}$ if $e^{xy^2} + \sin^{-1}2x = \csc y$.

10

$$\underbrace{e^{xy^2}}_{1}(\underbrace{y^2}_{1} + \underbrace{2xyy'}_{1\frac{1}{2}}) + \underbrace{\frac{2}{\sqrt{1-4x^2}}}_{1} = \underbrace{-(\csc y \cot y)y'}_{1\frac{1}{2}}$$

$$y^2e^{xy^2} + 2xye^{xy^2}y' + \frac{2}{\sqrt{1-4x^2}} = -(\csc y \cot y)y'$$

$$\underbrace{(2xye^{xy^2} + \csc y \cot y)y'}_{1\frac{1}{2}} = \underbrace{-\left(y^2e^{xy^2} + \frac{2}{\sqrt{1-4x^2}}\right)}_{1\frac{1}{2}}$$

$$y' = \left.-\frac{y^2e^{xy^2} + \frac{2}{\sqrt{1-4x^2}}}{2xye^{xy^2} + \csc y \cot y}\right]^{1} = -\frac{y^2e^{xy^2}\sqrt{1-4x^2} + 2}{(2xye^{xy^2} + \csc y \cot y)\sqrt{1-4x^2}}$$

Prove that the families of curves $y = cx^3$ and $x^2 + 3y^2 = k$ are orthogonal. **SCORE: ___ / 12 POINTS**

$$\frac{dy}{dx} = 3cx^2 \quad 1$$

$$2x + 6y\frac{dy}{dx} = 0 \quad 2$$

$$\frac{dy}{dx} = \frac{-x}{3y} \quad 2$$

$$3cx^2 \cdot \frac{-x}{3y} = -\frac{cx^3}{y} = -\frac{y}{y} = -1$$

3 2 2

SCORE: ___ / 12 POINTS

Determine whether Rolle's Theorem or the Mean Value Theorem (or both, or neither) apply to the function $f(x) = x^3 - 4x - 2$ on the interval $[-1, 0]$. If either theorem applies, find the corresponding value of c guaranteed by that theorem.

SINCE f IS A POLYNOMIAL, 1 f IS CONT. ON $[-1,0]$ 1 AND DIFF. ON $(-1,0)$, 1

SO MVT APPLIES 1

SINCE $f(-1) = 1$ AND $f(0) = -2$, SO $f(-1) \neq f(0)$, 1

SO ROLLE'S TH'M DOES NOT APPLY 1

$$f'(c) = \frac{f(0) - f(-1)}{0 - -1} \quad 2$$

$$3c^2 - 4 = -3 \quad 2$$

$$3c^2 = 1$$

$$c = \pm\frac{1}{\sqrt{3}} \quad 1 \qquad c = -\frac{1}{\sqrt{3}} \in (-1, 0) \quad 1$$

The velocity of a ball, in feet per second, hit by a baseball bat which weighs m pounds is given by the function $u(m)$. Interpret the statement $u'(3) = 1.2$. **SCORE: ___ / 6 POINTS**

Specify the units of all relevant numbers. Do NOT use the words *instantaneous*, *slope* or *derivative*, nor the phrase *rate of change*.

IF A BAT WEIGHS 3 POUNDS, 2

THE VELOCITY OF A BALL HIT BY THE BAT INCREASES $1.2\ \frac{FT}{SEC}$ 2

PER POUND OF ADDITIONAL WEIGHT ADDED TO THE BAT 2

Prove that $\lim_{x \to -2} (3-2x) = 7$ using the precise definition of a limit. **SCORE: ___ / 12 POINTS**

GIVEN ANY $\varepsilon > 0$ 2

LET $\delta = \frac{\varepsilon}{2}$ 4

IF $0 < |x-(-2)| < \delta$ 2

THEN $0 < |x+2| < \frac{\varepsilon}{2}$

$0 < |-2||x+2| < \varepsilon$ 2

$0 < |-2x-4| < \varepsilon$

$0 < |(3-2x)-7| < \varepsilon$ 2 QED

If $f(3) = -2$, $f'(3) = 4$ and $g(x) = \dfrac{1+x^2}{f(x)}$, find the equation of the tangent line to $y = g(x)$ at $x = 3$. **SCORE: ___ / 10 POINTS**

$g(3) = \frac{1+3^2}{f(3)} = \frac{10}{-2} = -5$ 2

$g'(x) = \dfrac{2xf(x) - (1+x^2)f'(x)}{(f(x))^2}$ 4

$g'(3) = \dfrac{2(3)(-2) - (1+3^2)4}{(-2)^2}$

$= \dfrac{-12-40}{4}$

$= -13$ 2

$y-(-5) = -13(x-3)$ 2

$y = -5 - 13(x-3)$

The position of an object, in meters, at time t seconds is given by $s(t) = t^2 \cos 2t$. **SCORE: ___ / 10 POINTS**
Find the acceleration of the object at time $t = \pi$.

$s'(t) = 2t\cos 2t - 2t^2 \sin 2t$ 3

$s''(t) = 2\cos 2t - 4t\sin 2t - 4t\sin 2t - 4t^2\cos 2t$ 5

$s''(\pi) = 2\cos 2\pi - 4\pi \sin 2\pi - 4\pi \sin 2\pi - 4\pi^2 \cos 2\pi$

$= 2 - 0 - 0 - 4\pi^2$

$= 2 - 4\pi^2 \ \frac{m}{s^2}$ 2