Math 1A (7:30am – 8:20am)
Midterm 3
Wed Nov 26, 2008

SCORE: ___ / 120 POINTS

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

ONLY SCIENTIFIC CALCULATORS ALLOWED

State the definition of "local maximum". **SCORE: ___ / 4 POINTS**

f HAS A LOCAL MAXIMUM AT $x=a$
IF $f(x) \le f(a)$ FOR ALL x IN AN OPEN INTERVAL CONTAINING a

State the definition of "inflection point". **SCORE: ___ / 4 POINTS**

f HAS AN INFLECTION POINT AT $x=a$
IF f IS CONCAVE UP FOR $x<a$ AND CONCAVE DOWN FOR $x>a$
OR " DOWN " UP "

State the Extreme Value Theorem. **SCORE: ___ / 4 POINTS**

IF f IS CONTINUOUS ON $[a,b]$
THEN f HAS BOTH A GLOBAL MAXIMUM AND MINIMUM ON $[a,b]$

Find the global extrema of $f(x) = \dfrac{x^2-5}{x+3}$ on $[-2, 2]$, and classify each as a maximum or minimum. **SCORE: ___ / 10 POINTS**

$$f'(x) = \frac{2x(x+3)-(x^2-5)}{(x+3)^2} = \frac{x^2+6x+5}{(x+3)^2} = \frac{(x+1)(x+5)}{(x+3)^2}$$

f' DNE AT $x=-3$, NOT IN $[-2,2]$
$f'=0$ AT $x=-1$, IN $[-2,2]$, AND $x=-5$, NOT IN $[-2,2]$

x	$f(x)$
-2	$\frac{-1}{1}=-1$
-1	$\frac{-4}{2}=-2$
2	$\frac{-1}{5}$

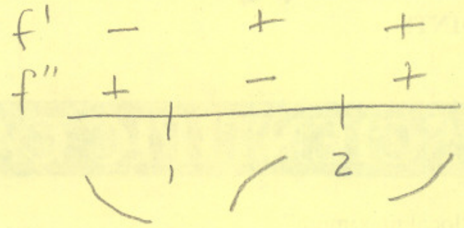
GLOBAL MAX $(2,-\frac{1}{5})$
MIN $(-1,-2)$

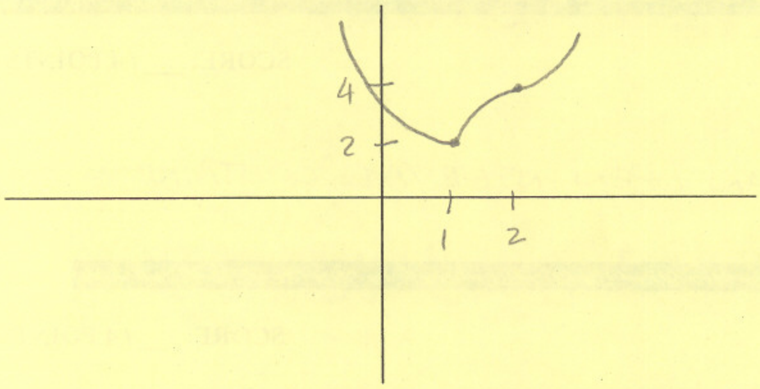
Sketch a graph of a function $f(x)$ which satisfies all the following conditions. **SCORE: ___ / 8 POINTS**

$f(1) = 2$, $f(2) = 4$, $f'(1)$ is undefined,

$f'(x) < 0$ if $x < 1$, $f'(x) > 0$ if $x > 1$,

$f''(x) > 0$ if $x < 1$ or $x > 2$, $f''(x) < 0$ if $1 < x < 2$





Evaluate the following limits. Show all relevant work. **SCORE: ___ / 14 POINTS**

[a] $\lim\limits_{x \to 1} x^{\left(\frac{1}{1-x}\right)}$

$\lim\limits_{x \to 1} \ln\left(x^{\frac{1}{1-x}}\right)$

$= \lim\limits_{x \to 1} \frac{1}{1-x} \ln x$

$= \lim\limits_{x \to 1} \frac{\ln x}{1-x} \quad \frac{0}{0}$

$= \lim\limits_{x \to 1} \frac{\frac{1}{x}}{-1} = \frac{1}{-1} = -1$

so $\lim\limits_{x \to 1} x^{\frac{1}{1-x}} = e^{-1}$

[b] $\lim\limits_{x \to 2} \frac{x^3 - 3x^2 + 4}{x^4 - 3x^2 - 20x + 36} \quad \frac{0}{0}$

$= \lim\limits_{x \to 2} \frac{3x^2 - 6x}{4x^3 - 6x - 20} \quad \frac{0}{0}$

$= \lim\limits_{x \to 2} \frac{6x - 6}{12x^2 - 6} = \frac{6}{42} = \frac{1}{7}$

Use an appropriate linear approximation to estimate $\tan^{-1} 0.9$. **SCORE: ___ / 10 POINTS**

$f(x) = \tan^{-1} x$

$x_0 = 1$

$L(x) = f(x_0) + f'(x_0)(x - x_0)$

$= \tan^{-1} 1 + \frac{1}{1+1^2}(x-1)$

$= \frac{\pi}{4} + \frac{1}{2}(x-1)$

$f(0.9) \approx L(0.9) = \frac{\pi}{4} + \frac{1}{2}(0.9-1) = \frac{\pi}{4} - 0.05$

Using Newton's method to approximate $\sqrt{2}$, find values for x_0, x_1 and x_2 in decimal. **SCORE: ___ / 10 POINTS**

NOTE: You do NOT need to complete Newton's method. You only need to give a reasonable value for x_0, then calculate x_1 and x_2.

$f(x) = x^2 - 2$

$x_0 = 1$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{-1}{2(1)} = \frac{3}{2}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{3}{2} - \frac{\frac{1}{4}}{2(\frac{3}{2})} = \frac{3}{2} - \frac{1}{12} = \frac{17}{12}$

Find the critical points of $f(x) = \dfrac{x^2 - 2}{x^4}$, and using the SECOND DERIVATIVE TEST, classify each as **SCORE: ___ / 10 POINTS**

a local maximum or minimum. **You must show the calculated values of the second derivative.**

$f(x) = x^{-2} - 2x^{-4}$

$f'(x) = -2x^{-3} + 8x^{-5}$ DNE AT $x = 0$, NOT IN DOMAIN

$= -2x^{-5}(x^2 - 4) = 0$ AT $x = \pm 2$, IN DOMAIN

$f''(x) = 6x^{-4} - 40x^{-6}$

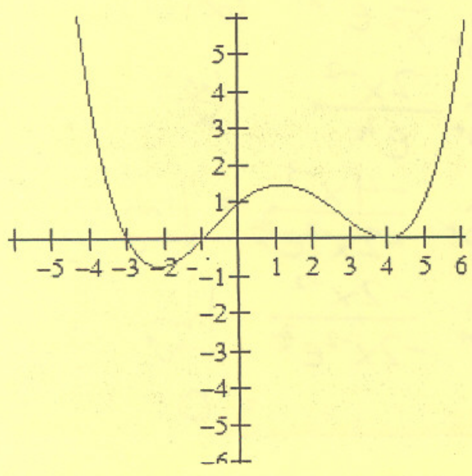
$f''(2) = \frac{6}{16} - \frac{40}{64} = \frac{3}{8} - \frac{5}{8} < 0$ $x = 2$ IS LOCAL MAX

$f''(-2) = \frac{6}{16} - \frac{40}{64} < 0$ $x = -2$ MAX

The graph below shows $f'(x)$. **IT IS NOT $f(x)$.** **SCORE: ___ / 10 POINTS**

Answer the following questions about the function $f(x)$.

$f'(x)$



[a] Write "YES" if you understand the instruction that this is **NOT** the graph of $f(x)$.

YES

[b] Estimate the interval(s) over which $f(x)$ is decreasing. $f'(x) < 0$

$[-3, -1]$

[c] Estimate the interval(s) over which $f(x)$ is concave up. $f'(x)$ INCR

$[-2, 1]$, $[4, \infty)$

[d] Estimate the x-coordinate(s) of the local maximum(a) of $f(x)$. $f'(x) > 0$ BECOMES $f'(x) < 0$

$x = -3$

Sketch a graph of $f(x) = e^{-\frac{2}{x}}$ using the full procedure discussed in class. **SCORE: ___ / 20 POINTS**

DOMAIN $x \neq 0$

DISCONTINUITY AT $x=0$

NO X-INT $\left(e^{-\frac{2}{x}} > 0\right)$

NO Y-INT ($x=0$ NOT IN DOMAIN)

$\lim\limits_{x\to 0^+} e^{-\frac{2}{x}} = \lim\limits_{x\to -\infty} e^x = 0$ (LIMIT POINT $(0,0)$)

$\lim\limits_{x\to 0^-} e^{-\frac{2}{x}} = \lim\limits_{x\to \infty} e^x = \infty$ (ONE SIDED VERTICAL ASYMPTOTE $x=0$)

$\lim\limits_{x\to \infty} e^{-\frac{2}{x}} = \lim\limits_{x\to 0^-} e^x = 1$ (HORIZONTAL ASYMPTOTE $y=1$)

$\lim\limits_{x\to -\infty} e^{-\frac{2}{x}} = \lim\limits_{x\to 0^+} e^x = 1$

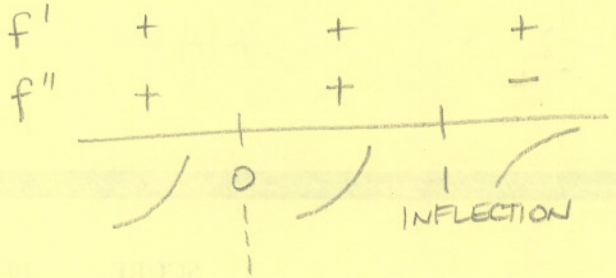
$f'(x) = \frac{2}{x^2} e^{-\frac{2}{x}}$ DNE AT $x=0$, NOT IN DOMAIN

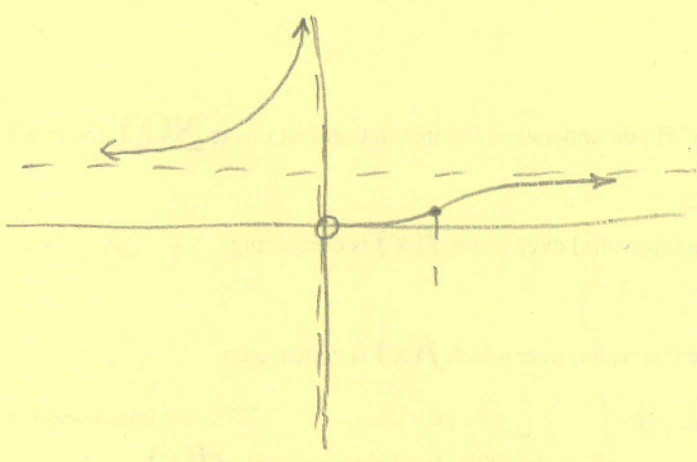
IS NEVER 0

$f''(x) = -\frac{4}{x^3} e^{-\frac{2}{x}} + \frac{2}{x^2}\left(\frac{2}{x^2} e^{-\frac{2}{x}}\right) = -\frac{4}{x^4} e^{-\frac{2}{x}}(x-1)$ DNE AT $x=0$, NOT IN DOMAIN

$= 0$ IF $x=1$

POINT $(1, e^{-2}) \approx (1, \frac{1}{8})$





NOT REQUIRED:

$\lim\limits_{x\to 0^+} f'(x)$

$= \lim\limits_{x\to 0^+} \frac{2}{x^2} e^{-\frac{2}{x}}$ $(\infty \cdot 0)$

$= \lim\limits_{x\to 0^+} \frac{2x^{-2}}{e^{\frac{2}{x}}}$ $\left(\frac{\infty}{\infty}\right)$

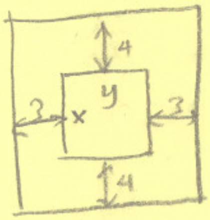
$= \lim\limits_{x\to 0^+} \frac{-4x^{-3}}{-2x^{-2} e^{\frac{2}{x}}} = \lim\limits_{x\to 0^+} \frac{2x^{-1}}{e^{\frac{2}{x}}}$ $\left(\frac{\infty}{\infty}\right)$

$= \lim\limits_{x\to 0^+} \frac{-2x^{-2}}{-2x^{-2} e^{\frac{2}{x}}} = \lim\limits_{x\to 0^+} \frac{1}{e^{\frac{2}{x}}}$ $\frac{1}{\infty}$

$= 0$

An advertisement consists of a rectangular printed region plus 3 cm margins on the side and 4 cm margins at top and bottom. If the area of the printed region is to be 300 cm^2, find the dimensions of the printed region that minimize the total area. **SHOW ALL WORK.**

SCORE: ___ / 16 POINTS



A = AREA

x, y = DIMENSIONS OF PRINTED REGION

$A = (x+8)(y+6)$

$xy = 300$

$y = \frac{300}{x}$

$A = (x+8)\left(\frac{300}{x}+6\right) \qquad x \in (0, \infty)$

$= 300 + \frac{2400}{x} + 6x + 48$

$= 348 + \frac{2400}{x} + 6x$

$A' = -\frac{2400}{x^2} + 6$ DNE AT $x=0$, NOT IN DOMAIN

$= 0$ IF $\frac{2400}{x^2} = 6$

$400 = x^2$

$x = 20$

$y = 15$ $\Rightarrow A = (28)(21) = 588$

$\lim_{x \to 0^+} A = \lim_{x \to \infty} A = \infty$

20cm × 15cm

☺ BONUS QUESTIONS ☺ ON OTHER SIDE