Math 1A (7:30am – 8:20am)
Quiz 2
Fri Oct 3, 2008

SCORE: ___ / 20 POINTS

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

Complete the following definition: 1 POINT IF 2 PARTS RIGHT **SCORE: ___ / 2 POINTS**

A function f is continuous at a point $x = a$ if 2 POINTS IF ALL 3 PARTS RIGHT

[1] $f(a)$ EXISTS

[2] $\lim_{x \to a} f(x)$ EXISTS

[3] $\lim_{x \to a} f(x) = f(a)$

State the Intermediate Value Theorem. **SCORE: ___ / 2 POINTS**

IF f IS CONTINUOUS ON $[a,b]$ AND $f(a) \neq f(b)$ $\frac{1}{2}$
AND W IS BETWEEN $f(a)$ AND $f(b)$, $\frac{1}{2}$
THEN THERE EXISTS c BETWEEN a AND b, $\frac{1}{2}$
SUCH THAT $f(c) = W$ $\frac{1}{2}$

Show that the function $f(x) = x^4 - 4x^2 + 1$ has a zero in the interval $[1, 2]$. **SCORE: ___ / 4 POINTS**

You must justify your argument properly as shown in class.

1 f IS CONTINUOUS ON $[1,2]$ SINCE f IS A POLYNOMIAL $\frac{1}{2}$

$f(1) = -2$

$f(2) = 1 \neq f(1)$ $\frac{1}{2}$

$f(1) \leq 0 \leq f(2)$ $\frac{1}{2}$

1 BY IVT FOR SOME $c \in (1,2)$, $\frac{1}{2}$
$f(c) = 0$

IE. f HAS A ZERO IN $[1,2]$

CONTINUED ON OTHER SIDE

Let $f(x)=\begin{cases} x^2+1 & \text{if } x\le 1 \\ 3-x^3 & \text{if } 1<x<3 \\ 1-7x & \text{if } x\ge 3 \end{cases}$.

SCORE: ___ / 12 POINTS

[a] Evaluate the following limits. If a limit does not exist, briefly explain why not.
Show enough work so that it is clear where your answer comes from.

[1] $\lim_{x\to 2} f(x)$

$\lim_{x\to 2} f(x) = \lim_{x\to 2}(3-x^3) = -5$ (1 ±)

[2] $\lim_{x\to 3} f(x)$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(3-x^3) = -24$ (1 ±)

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(1-7x) = -20$ (1 ±)

$\lim_{x\to 3^+} f(x) \neq \lim_{x\to 3^-} f(x)$ (1) so $\lim_{x\to 3} f(x)$ DNE (1)

[3] $\lim_{x\to 1^-} f(x)$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x^2+1) = 2$ (1 ±)

[b] Find all discontinuities of $f(x)$. For each discontinuity, state whether the discontinuity is removable.

f IS DISCONTINUOUS AT $x=3$ (1)

SINCE $\lim_{x\to 3} f(x)$ DNE

FOR THE SAME REASON,

THE DISCONTINUITY IS NOT REMOVABLE (1)

[c] Find all intervals on which $f(x)$ is continuous.

$(-\infty, 3)$ (1), $(3, \infty)$ (1)