

What day of the month is your birthday? ___

What are the last 2 digits of your address? ___

What are the last 2 digits of your zip code? ___

What are the last 2 digits of your social security number? ___

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]

State the Intermediate Value Theorem.

SCORE: ___ / 2 POINTS

IF f IS CONTINUOUS ON $[a, b]$ AND $f(a) \neq f(b)$ $\frac{1}{2}$
 AND w IS BETWEEN $f(a)$ AND $f(b)$, $\frac{1}{2}$
 THEN THERE EXISTS c BETWEEN a AND b , $\frac{1}{2}$
 SUCH THAT $f(c) = w$ $\frac{1}{2}$

Complete the following definition:

SCORE: ___ / 2 POINTS

A function f is continuous at a point $x = a$ if

1 POINT IF 2 PARTS CORRECT
 2 POINTS IF ALL 3 PARTS CORRECT

- [1] $f(a)$ EXISTS
 [2] $\lim_{x \rightarrow a} f(x)$ EXISTS
 [3] $\lim_{x \rightarrow a} f(x) = f(a)$

Show that the function $f(x) = x^4 - 6x^2 + 1$ has a zero in the interval $[-1, 0]$.

SCORE: ___ / 4 POINTS

You must justify your argument properly as shown in class.

1 f IS CONTINUOUS ON $[-1, 0]$ SINCE f IS A POLYNOMIAL $\frac{1}{2}$
 $f(-1) = -4$
 $f(0) = 1 \neq f(-1)$ $\frac{1}{2}$
 $f(-1) \leq 0 \leq f(0)$ $\frac{1}{2}$
 1 BY IVT, FOR SOME $c \in (-1, 0)$, $\frac{1}{2}$
 $f(c) = 0$
 IE. f HAS A ZERO IN $[-1, 0]$

$$\text{Let } f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 1 \\ 3 + x^3 & \text{if } 1 < x < 3 \\ 9 + 7x & \text{if } x \geq 3 \end{cases}$$

SCORE: ___ / 12 POINTS

- [a] Evaluate the following limits. If a limit does not exist, briefly explain why not.
Show enough work so that it is clear where your answer comes from.

[1] $\lim_{x \rightarrow 1} f(x)$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x^2 + 1) = 2$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (3 + x^3) = 4$$

$$\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x) \quad \text{so } \lim_{x \rightarrow 1} f(x) \text{ DNE}$$

[2] $\lim_{x \rightarrow 3^+} f(x)$

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (9 + 7x) = 30$$

[3] $\lim_{x \rightarrow 4} f(x)$

$$\lim_{x \rightarrow 4} f(x) = \lim_{x \rightarrow 4} (9 + 7x) = 37$$

- [b] Find all discontinuities of $f(x)$. For each discontinuity, state whether the discontinuity is removable.

f IS DISCONTINUOUS AT $x = 1$

SINCE $\lim_{x \rightarrow 1} f(x) \text{ DNE}$

FOR THE SAME REASON,

THE DISCONTINUITY IS NOT REMOVABLE

- [c] Find all intervals on which $f(x)$ is continuous.

$$(-\infty, 1), (1, \infty)$$