

SCORE: ___ / 20 POINTS

What day of the month is your birthday? ___

What are the last 2 digits of your address? ___

What are the last 2 digits of your zip code? ___

What are the last 2 digits of your social security number? ___

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]**NO CALCULATORS ALLOWED**

Find the derivatives of the following functions.

SCORE: ___ / 8 POINTS

FOR 1 BONUS POINT EACH, SIMPLIFY YOUR ANSWERS.

[a] $f(x) = \frac{x^2}{\sqrt{x^3+1}}$

$$\begin{aligned}
 f'(x) &= \frac{2x \sqrt{x^3+1} - \frac{1}{2} x^2 (x^3+1)^{-\frac{1}{2}} 3x^2}{x^3+1} \\
 &= \frac{2x(x^3+1) - \frac{3}{2} x^4}{(x^3+1)^{\frac{3}{2}}} \\
 &= \frac{4x(x^3+1) - 3x^4}{2(x^3+1)^{\frac{3}{2}}} \\
 &= \frac{x^4 + 4x}{2(x^3+1)^{\frac{3}{2}}}
 \end{aligned}$$

[b] $f(x) = (x+2)^5 (x^2+4)^4$

$$\begin{aligned}
 f'(x) &= 5(x+2)^4 (x^2+4)^4 + (x+2)^5 4(x^2+4)^3 (2x) \\
 &= (x+2)^4 (x^2+4)^3 (5(x^2+4) + 8x(x+2)) \\
 &= (x+2)^4 (x^2+4)^3 (13x^2 + 16x + 20)
 \end{aligned}$$

Find the derivative of $\frac{1}{1+[f(x)]^2}$, where $f(x)$ is an unspecified differentiable function

SCORE: ___ / 4 POINTS

$$\begin{aligned}
 \text{DERIVATIVE} &= -[1+[f(x)]^2]^{-2} \cdot 2f(x)f'(x) \\
 &= \frac{-2f(x)f'(x)}{[1+[f(x)]^2]^2}
 \end{aligned}$$

Find the derivative of $(3x^3 + 7x - 6)(8x^2 - x - 5)(4x^3 - 2x^2 + 9)$.

SCORE: ___ / 3 POINTS

DO NOT SIMPLIFY YOUR ANSWER.

$$\begin{aligned} & (9x^2 + 7)(8x^2 - x - 5)(4x^3 - 2x^2 + 9) \\ & + (3x^3 + 7x - 6)(16x - 1)(4x^3 - 2x^2 + 9) \\ & + (3x^3 + 7x - 6)(8x^2 - x - 5)(12x^2 - 4x) \end{aligned}$$

The following table gives the function and derivative values of two differentiable functions $f(x)$ and $g(x)$ for various input values.

SCORE: ___ / 5 POINTS

x	0	1	2	3	4	5
$f(x)$	3	0	5	1	2	4
$f'(x)$	5	2	3	4	6	1
$g(x)$	1	5	4	2	0	3
$g'(x)$	1	4	5	6	3	2

[a] If $k(x) = g(f(x))$, find the slope of the tangent line to $y = k(x)$ at $x = 0$.

$$\begin{aligned} k'(x) &= g'(f(x)) \cdot f'(x) \\ k'(0) &= g'(f(0)) \cdot f'(0) \\ &= g'(3) \cdot 5 \\ &= 6 \cdot 5 \\ &= 30 \end{aligned}$$

[b] If $w(x) = \sqrt{x}f(x)$, find the slope of the tangent line to $y = w(x)$ at $x = 4$.

$$\begin{aligned} w'(x) &= \frac{1}{2}x^{-\frac{1}{2}}f(x) + \sqrt{x}f'(x) \\ w'(4) &= \frac{1}{2}4^{-\frac{1}{2}}f(4) + \sqrt{4}f'(4) \\ &= \frac{1}{4} \cdot 2 + 2 \cdot 6 \\ &= 3\frac{1}{2} \end{aligned}$$