

SCORE: ____ / 10 POINTS

NO CALCULATORS ALLOWED

2.71828182845904523536028747135266249775724709369995957496696762772407663035354759457138217852516642742746639193200

Using determinants, determine if $\begin{bmatrix} -2^+ & 1 & 3 & -1 \\ 0^- & 4 & 2 & 1 \\ -4^+ & 1 & 5 & -3 \\ 2^- & 0^+ & 0^- & 3^+ \end{bmatrix}$ has an inverse.

SCORE: ____ / 3 POINTS

$$-2 \begin{vmatrix} 1 & 3 & -1 \\ 4 & 2 & 1 \\ 1 & 5 & -3 \end{vmatrix} = -2 \begin{vmatrix} 1 & 3 \\ 4 & 2 \\ 1 & 5 \end{vmatrix}$$

$$+3 \begin{vmatrix} -2 & 1 & 3 \\ 0 & 4 & 2 \\ -4 & 1 & 5 \end{vmatrix} = +3 \begin{vmatrix} -2 & 1 & 3 \\ 0 & 4 & 2 \\ -4 & 1 & 5 \end{vmatrix}$$

$\frac{1}{2}$ POINT EACH

$$= -2(-6+3-20 - -2-5--36) + 3(-40-8+0 - -48--4-0)$$

$$= -2(10) + 3(4) = -8 \neq 0 \quad \text{INVERSE EXISTS}$$

2.71828182845904523536028747135266249775724709369995957496696762772407663035354759457138217852516642742746639193200

If A is a 2×2 matrix and $\det A = 5$, find $\det 3A$. Show your work.

SCORE: ____ / 2 POINTS

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$ad - bc = 5$$

$$3A = \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix}$$

$$= 9ad - 9bc$$

$$= 9(ad - bc) = 45$$

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Write the partial fraction decomposition of the following.

SCORE: ____ / 5 POINTS

[a] $\frac{x^2-1}{x^3-4x^2+4x} = \frac{x^2-1}{x(x-2)^2}$

$$= \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x^2-1 = A(x-2)^2 + Bx(x-2) + Cx$$

$x=0 \quad -1 = 4A \Rightarrow A = -\frac{1}{4}$

$x=2 \quad 3 = 2C \Rightarrow C = \frac{3}{2}$

COEF OF $x^2 \quad 1 = A+B \Rightarrow B = 1-A = \frac{5}{4}$

CHECK $x=1$

$$0 = -\frac{1}{4}(1) + \frac{5}{4}(-1) + \frac{3}{2}(1)$$

$$= -\frac{1}{4} - \frac{5}{4} + \frac{3}{2}$$

$$= 0 \quad \checkmark$$

$$\frac{-\frac{1}{4}}{x} + \frac{\frac{5}{4}}{x-2} + \frac{\frac{3}{2}}{(x-2)^2}$$

[b] $\frac{2x^2+5}{x^3+9x} = \frac{2x^2+5}{x(x^2+9)} = \frac{A}{x} + \frac{Bx+C}{x^2+9}$

$$2x^2+5 = A(x^2+9) + Bx^2+Cx$$

$x=0 \quad 5 = 9A \Rightarrow A = \frac{5}{9}$

COEF OF $x^2 \quad 2 = A+B \Rightarrow B = 2-A = \frac{13}{9}$

COEF OF $x \quad 0 = C$

CHECK $x=1$

$$7 = \frac{5}{9}(10) + \frac{13}{9}(1)$$

$$= \frac{50}{9} + \frac{13}{9}$$

$$= \frac{63}{9}$$

$$= 7 \quad \checkmark$$

$$\frac{\frac{5}{9}}{x} + \frac{\frac{13}{9}x}{x^2+9}$$

SCORE: ____ / 10 POINTS

NO CALCULATORS ALLOWED

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328

Using determinants, determine if $\begin{bmatrix} -2 & 0 & 3 & -1 \\ 0 & 4 & 2 & 1 \\ -4 & 1 & 5 & -3 \\ 2 & 0 & -1 & 3 \end{bmatrix}$ has an inverse.

SCORE: ____ / 3 POINTS

$$\begin{array}{c} 4 \begin{vmatrix} -2 & 3 & -1 \\ -4 & 5 & -3 \\ 2 & -1 & 3 \end{vmatrix} - 2 \begin{vmatrix} -2 & 3 \\ -4 & 5 \end{vmatrix} - 1 \begin{vmatrix} -2 & 3 & -1 \\ 0 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix} - 2 \begin{vmatrix} -2 & 3 \\ 0 & 2 \end{vmatrix} \end{array}$$

$\frac{1}{2}$ POINT EACH

$$= 4(-30 - 18 - 4 - 10 - 6 - 36) - (-12 + 6 + 0 - 4 - 2 - 0) \\ = 4(0) - (-4) = 4 \neq 0 \quad \text{INVERSE EXISTS}$$

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328

If A is a 2×2 matrix and $\det A = 5$, find $\det 3A$. Show your work.

SCORE: ____ / 2 POINTS

SEE OTHER KEY

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328

Write the partial fraction decomposition of the following.

SCORE: ____ / 5 POINTS

[a] $\frac{2x^2 + 5}{x^3 + 9x}$

[b] $\frac{x^2 - 1}{x^3 - 4x^2 + 4x}$

SEE OTHER KEY