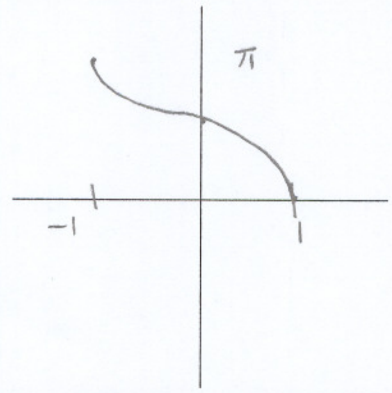
Sketch the graph of $y = \cos^{-1} x$, and state the domain and range. **SCORE: ___ / 8 POINTS**

DOMAIN = $[-1, 1]$ 1 1 1

RANGE = $[0, \pi]$ 1 1



POSITION 2

SHAPE 2

Evaluate the following expressions. If a value does not exist, write UNDEFINED. **SCORE: ___ / 9 POINTS**

(a) $\cos^{-1}(0) = \frac{\pi}{2}$ 1

(b) $\arctan(-\sqrt{3}) = -\frac{\pi}{3}$ 1

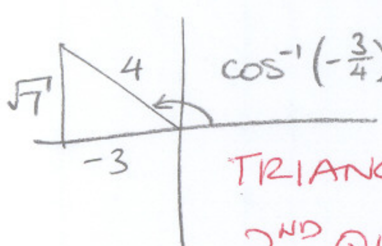
(c) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ 1

(d) $\sin\left(\arcsin\left(\frac{3\pi}{2}\right)\right) =$ UNDEFINED 2

(e) $\tan^{-1}\left(\tan\left(\frac{5\pi}{6}\right)\right) = -\frac{\pi}{6}$ 2

(f) $\arccos\left(\cos\left(\frac{2\pi}{9}\right)\right) = \frac{2\pi}{9}$ 2

Find the exact value of $\tan\left(\cos^{-1}\left(-\frac{3}{4}\right)\right)$. **SCORE: ___ / 6 POINTS**



$-\frac{\sqrt{7}}{3}$ 3

TRIANGLE 2

2^{ND} QUADRANT 1

Find the exact solutions of the equation $\sqrt{3} + 2\sin 2x = 0$ in the interval $[0, 2\pi)$. **SCORE: ___ / 10 POINTS**

$\sin 2x = -\frac{\sqrt{3}}{2}$ 2

$0 \le x < 2\pi$

$0 \le 2x < 4\pi$ (TWICE AROUND) 1

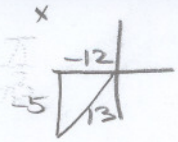
$1\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$

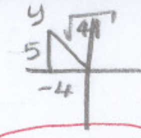
$2x = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$ $\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

1 1 $\frac{1}{2}$ $\frac{1}{2}$

If $\pi < x < \frac{3\pi}{2}$ and $\frac{\pi}{2} < y < \pi$, and $\sin x = -\frac{5}{13}$ and $\tan y = -\frac{5}{4}$, find the values of the following expressions.





SCORE: ___ / 14 POINTS

(a) $\tan 2y = \frac{2\tan y}{1-\tan^2 y}$ 1

$= \frac{2\left(-\frac{5}{4}\right)}{1-\left(-\frac{5}{4}\right)^2}$ 2

$= \frac{-\frac{5}{2}}{1-\frac{25}{16}}$

$= \frac{-5}{\cancel{2}} \cdot \frac{\cancel{-16}\ -8}{9}$

$= \frac{40}{9}$ 2

(b) $\sin \frac{y}{2} = \pm\sqrt{\frac{1-\cos y}{2}}$ 1

$= \pm\sqrt{\frac{1 - -\frac{4}{\sqrt{41}}}{2}}$ 2

$= \pm\sqrt{\frac{\sqrt{41}+4}{2\sqrt{41}}}$ $\frac{1}{2}$

$= \pm\sqrt{\frac{41+4\sqrt{41}}{82}}$ $\frac{1}{2}$

$= +\sqrt{\frac{41+4\sqrt{41}}{82}}$ 1

$\frac{\pi}{2} < y < \pi$

$\frac{\pi}{4} < \frac{y}{2} < \frac{\pi}{2}$

Q_1

$\sin \frac{y}{2} > 0$

(c) $\cos(x-y) = \cos x \cos y + \sin x \sin y$ 1

$= \frac{-12}{13} \cdot \frac{-4}{\sqrt{41}} + \frac{-5}{13} \cdot \frac{5}{\sqrt{41}}$ 2

$= \frac{23}{13\sqrt{41}}$ 1

Find the exact solutions of the equation $\cos 2x = -2\sin x$ in the interval $[0, 2\pi)$.
Your answer might involve an inverse trigonometric function.

SCORE: ___ / 12 POINTS

2 $1 - 2\sin^2 x = -2\sin x$

$0 = 2\sin^2 x - 2\sin x - 1$ 2

2 $\sin x = \frac{2 \pm \sqrt{4 - -8}}{4} = \frac{2\pm\sqrt{12}}{4} = \frac{2\pm 2\sqrt{3}}{4}$

$= \frac{1\pm\sqrt{3}}{2}$ 1

$x = \sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right) + 2\pi$ 1

2

or

$\pi - \sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)$ 1

$\frac{1+\sqrt{3}}{2} > 1$ 1

so $\sin x = \frac{1-\sqrt{3}}{2} < 0$

Math 49B (8:30am – 9:20am)
Midterm 1
Thu Oct 16, 2008

SCORE: ___ / 120 POINTS

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

CALCULATOR ALLOWED ON THIS SECTION

Verify the identity $\dfrac{1}{\sec x - \tan x} + \dfrac{1+\csc x}{\cot x} = \dfrac{2\cos x}{1-\sin x}$. **SCORE: ___ / 12 POINTS**

SOLUTIONS WILL VARY

$$\frac{1}{\sec x - \tan x} + \frac{1+\csc x}{\cot x}$$

$$= \frac{1}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}} + \frac{1 + \frac{1}{\sin x}}{\frac{\cos x}{\sin x}}$$

$$= \frac{\cos x}{1-\sin x} + \frac{\sin x + 1}{\cos x}$$

$$= \frac{\cos^2 x + (\sin x + 1)(1-\sin x)}{(1-\sin x)\cos x}$$

$$= \frac{\cos^2 x + 1 - \sin^2 x}{(1-\sin x)\cos x}$$

$$= \frac{\cos^2 x + \cos^2 x}{(1-\sin x)\cos x}$$

$$= \frac{2\cos^{\cancel{2}} x}{(1-\sin x)\cancel{\cos x}}$$

$$= \frac{2\cos x}{1-\sin x}$$

QED

Write an algebraic expression that is equivalent to $\cos(2\tan^{-1} x)$, where $x > 0$. Simplify your answer. **SCORE: ___ / 9 POINTS**

LET $\theta = \tan^{-1} x$ 2 $\Longrightarrow \tan\theta = x$ AND $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

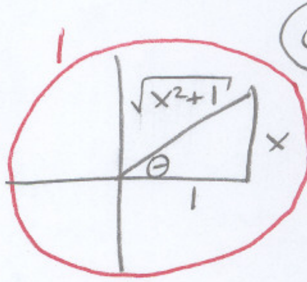
WANT $\cos 2\theta$ 1 Q_1 or Q_4

$= \cos^2\theta - \sin^2\theta$ 2

$= \left(\frac{1}{\sqrt{x^2+1}}\right)^2 - \left(\frac{x}{\sqrt{x^2+1}}\right)^2$ 2

$= \dfrac{1-x^2}{x^2+1}$ 1

1



If $\tan x = 2$ and $\csc x < 0$, **use identities** to find the exact values of $\sin x$ and $\sec x$. **SCORE: ___ / 9 POINTS**

DO NOT USE TRIANGLES OR QUADRANTS. DECIMAL APPROXIMATIONS ARE NOT ACCEPTABLE.

1 $\cot x = \dfrac{1}{\tan x} = \dfrac{1}{2}$ $\frac{1}{2}$

$1\frac{1}{2}$ $\csc^2 x = 1 + \cot^2 x$

$= 1 + \left(\frac{1}{2}\right)^2$

$= \frac{5}{4}$

$\csc x = -\dfrac{\sqrt{5}}{2}$ 1

1 $\sin x = \dfrac{1}{\csc x} = -\dfrac{2}{\sqrt{5}}$ 1

$\tan x = \dfrac{\sin x}{\cos x}$

$\tan x = \sin x \sec x$

2 $\sec x = \tan x \csc x$

$= 2 \cdot \left(-\frac{\sqrt{5}}{2}\right)$

$= -\sqrt{5}$ 1

Use the trigonometric substitution $x = 4\csc\theta$ to write $\sqrt{9x^2 - 144}$ as a trigonometric function of θ, where $0 < \theta < \dfrac{\pi}{2}$. **SCORE: ___ / 5 POINTS**

$\sqrt{9(4\csc\theta)^2 - 144}$ 1

$= \sqrt{144\csc^2\theta - 144}$ 1

$= 12\sqrt{\csc^2\theta - 1}$ 1

$= 12\sqrt{\cot^2\theta}$ 1

$= 12\cot\theta$ 1