Math 49B (9:30am – 10:20am)
Midterm 2
Fri Nov 14, 2008

SCORE: ___ / 120 POINTS

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO MATRIX-CAPABLE OR GRAPHING CALCULATORS ALLOWED ON THIS MIDTERM

Test $r = 1 + \cos 2\theta$ for symmetry with respect to $\theta = \frac{\pi}{2}$. **SCORE: ___ / 6 POINTS**

$(r, \pi - \theta)$

$r = 1 + \cos 2(\pi - \theta)$

$= 1 + \cos(2\pi - 2\theta)$

$= 1 + \cos 2\pi \cos 2\theta + \sin 2\pi \sin 2\theta = 1 + \cos 2\theta$ ✓

($\cos 2\pi = 1$, $\sin 2\pi = 0$)

SYMMETRIC

Use an inverse matrix to solve $\begin{matrix} 5x - 7y = 11 \\ 4x - 3y = 2 \end{matrix}$. **SCORE: ___ / 8 POINTS**

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5(-3)-4(-7)} \begin{bmatrix} -3 & 7 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

$$= \frac{1}{-15+28} \begin{bmatrix} -33+14 \\ -44+10 \end{bmatrix}$$

$$= \frac{1}{13} \begin{bmatrix} -19 \\ -34 \end{bmatrix} = \begin{bmatrix} -19/13 \\ -34/13 \end{bmatrix}$$

$x = -19/13$

$y = -34/13$

Find the standard form of $\left(2cis\frac{3\pi}{4}\right)^5$. **SCORE: ___ / 4 POINTS**

$2^5 \operatorname{cis} 5\left(\frac{3\pi}{4}\right)$

$= 32 \operatorname{cis} \frac{15\pi}{4}$ (COTERMINAL WITH $-\frac{\pi}{4}$)

$= 32\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = 16\sqrt{2} - 16\sqrt{2}i$

A point has polar coordinates $\left(4, \frac{\pi}{7}\right)$. **SCORE: ___ / 4 POINTS**

[a] Find another polar representation of the point using a negative value of r. $\left(-4, \frac{\pi}{7} + \pi\right) = \left(-4, \frac{8\pi}{7}\right)$

[b] Find another polar representation of the point using a negative value of θ. $\left(4, \frac{\pi}{7} - 2\pi\right) = \left(4, -\frac{13\pi}{7}\right)$

Math 49B (9:30am – 10:20am)
Midterm 2
Fri Nov 14, 2008

SCORE: ___ / 120 POINTS

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO MATRIX-CAPABLE OR GRAPHING CALCULATORS ALLOWED ON THIS MIDTERM

Test $r = 1 + \cos 2\theta$ for symmetry with respect to the pole. **SCORE: ___ / 6 POINTS**

$(r, \pi+\theta)$

$r = 1+\cos 2(\pi+\theta)$

$= 1+\cos(2\pi+2\theta)$ SYMMETRIC

$= 1+\cos 2\pi\cos 2\theta - \sin 2\pi \sin 2\theta = 1+\cos 2\theta$ ✓

Use an inverse matrix to solve $\begin{matrix} 5x-7y=11 \\ 4x-3y=2 \end{matrix}$. **SCORE: ___ / 8 POINTS**

SEE OTHER KEY

Find the standard form of $\left(2cis\frac{2\pi}{3}\right)^5$. **SCORE: ___ / 4 POINTS**

$2^5 \text{ cis } 5\left(\frac{2\pi}{3}\right)$

$= 32 \text{ cis } \frac{10\pi}{3}$ (COTERMINAL WITH $\frac{4\pi}{3}$)

$= 32\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -16 - 16\sqrt{3}i$

A point has polar coordinates $\left(2, \frac{\pi}{5}\right)$. **SCORE: ___ / 4 POINTS**

[a] Find another polar representation of the point using a negative value of r. $(-2, \frac{\pi}{5}+\pi) = (-2, \frac{6\pi}{5})$

[b] Find another polar representation of the point using a negative value of θ. $(2, \frac{\pi}{5}-2\pi) = (2, -\frac{9\pi}{5})$

Find parametric equations for the circle with a diameter with endpoints $(2,-7)$ and $(-6,-1)$. **SCORE: ___ / 8 POINTS**

CENTER $= \left(\frac{2+-6}{2}, \frac{-7+-1}{2}\right) = (-2,-4)$

RADIUS $= \frac{1}{2}\sqrt{(2--6)^2+(-7--1)^2}$

$= \frac{1}{2}\sqrt{64+36} = \frac{1}{2}\sqrt{100} = 5$

$x = -2+5\cos t$

$y = -4+5\sin t$

Find the zeros of $r = 1-2\sin 2\theta$. **SCORE: ___ / 10 POINTS**

$1-2\sin 2\theta = 0$

$\sin 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{6}+2n\pi, \frac{5\pi}{6}+2n\pi$

$\theta = \frac{\pi}{12}+n\pi, \frac{5\pi}{12}+n\pi$

Find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & -4 \\ 3 & -3 & 7 \end{bmatrix}$. **SCORE: ___ / 8 POINTS**

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ -2 & 3 & -4 & 0 & 1 & 0 \\ 3 & -3 & 7 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ R_2+2R_1 \\ R_3+(-3)R_1 \end{matrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right] R_1+(-2)R_3$$

R_1+R_2

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 7 & 0 & -2 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 9 & 1 & -2 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right]$$

INVERSE

Convert the rectangular equation $y = x^2 + x$ to polar form and **SIMPLIFY**. **SCORE: ___ / 10 POINTS**

$r\sin\theta = r^2\cos^2\theta + r\cos\theta$

$\sin\theta = r\cos^2\theta + \cos\theta$

$\sin\theta - \cos\theta = r\cos^2\theta$

$r = \frac{\sin\theta - \cos\theta}{\cos^2\theta}$

Write an augmented matrix for the following problem. **You must state clearly what your unknowns represent.** **SCORE: ___ / 8 POINTS**

DO NOT SOLVE THE SYSTEM OF EQUATIONS.

The tip jar at Give Pizza Chance contains $12.50 in dimes, quarters and dollar coins. There are 35 coins altogether, and the value of all the quarters combined is twice the value of all the dimes combined. How many of each type of coin are in the jar ?

d = # dimes
q = # quarters
c = # dollar coins

$$d + q + c = 35$$
$$0.1d + 0.25q + c = 12.5$$
$$0.25q = 2(0.1d)$$
$$\downarrow$$
$$-0.2d + 0.25q = 0$$

$$\begin{bmatrix} 1 & 1 & 1 & 35 \\ 0.1 & 0.25 & 1 & 12.5 \\ -0.2 & 0.25 & 0 & 0 \end{bmatrix}$$

Find the trigonometric form of $-3+\sqrt{3}i$. **SCORE: ___ / 4 POINTS**

$$r = \sqrt{(-3)^2 + (\sqrt{3})^2} = \sqrt{3+9} = \sqrt{12} = 2\sqrt{3}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-3}\right) + \pi = -\frac{\pi}{6} + \pi = \frac{5\pi}{6}$$

$$2\sqrt{3}\,\text{cis}\,\frac{5\pi}{6}$$

Use **Gauss-Jordan elimination** to solve the system $\begin{aligned} 2x - y - z &= 0 \\ x + y - 5z &= 3 \\ x - 2z &= 1 \end{aligned}$. **SCORE: ___ / 10 POINTS**

$$\left[\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ 1 & 1 & -5 & 3 \\ 1 & 0 & -2 & 1 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 1 & 1 & -5 & 3 \\ 2 & -1 & -1 & 0 \end{array}\right] \begin{array}{l} R_2 + (-1)R_1 \\ R_3 + (-2)R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & -1 & 3 & -2 \end{array}\right] \quad R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x - 2z = 1 \Rightarrow x = 2z + 1$
$y - 3z = 2 \Rightarrow y = 3z + 2$
$z = z$

$(2z+1, 3z+2, z)$

Convert the polar equation $r = \dfrac{6}{3-2\cos\theta}$ to rectangular form and **SIMPLIFY**. **SCORE: ___ / 10 POINTS**

$r = \dfrac{6}{3-\frac{2x}{r}}$

$3r - 2x = 6$

$3r = 2x + 6$

$3\sqrt{x^2+y^2} = 2x+6$

$\rightarrow 9(x^2+y^2) = 4x^2 + 24x + 36$

$5x^2 + 9y^2 - 24x - 36 = 0$

Consider the conic with polar equation $r = \dfrac{10}{2-5\sin\theta} \cdot \dfrac{\frac{1}{2}}{\frac{1}{2}} = \dfrac{5}{1-\frac{5}{2}\sin\theta}$ **SCORE: ___ / 10 POINTS**

[a] Identify the type of conic.

$e = \frac{5}{2} > 1$

HYPERBOLA

[b] Find the rectangular equation of the directrix of the conic.

$ep = 5$

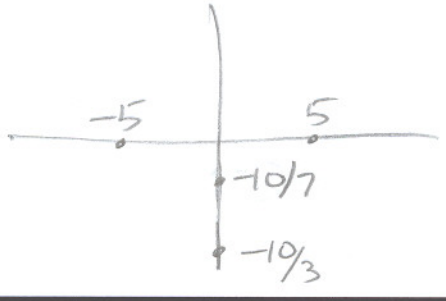
$\frac{5}{2}p = 5$

$p = 2$ HORIZONTAL BELOW POLE

$y = -2$

[c] Find all foci of the conic.

θ	r
0	5
$\frac{\pi}{2}$	$-10/3$
π	5
$\frac{3\pi}{2}$	$10/7$



$\frac{-10}{7} + \frac{-10}{3} = \frac{-100}{21}$

$(0, \frac{-100}{21})$ $(0, 0)$

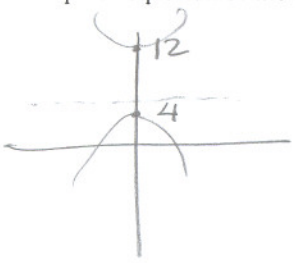
Find and **SIMPLIFY** a rectangular equation for the curve with parametric equations $x = 1-2t$, $y = \dfrac{t}{1+t}$. **SCORE: ___ / 8 POINTS**

$t = \dfrac{1-x}{2}$

$y = \dfrac{\frac{1-x}{2}}{1+\frac{1-x}{2}} \cdot \dfrac{2}{2}$

$y = \dfrac{1-x}{2+1-x} = \dfrac{1-x}{3-x}$

Find the polar equation of the hyperbola with the pole as a focus, and vertices at $\left(4, \frac{\pi}{2}\right)$ and $\left(-12, \frac{3\pi}{2}\right)$. **SCORE: ___ / 12 POINTS**



$r = \frac{ep}{1+e\sin\theta}$

$4 = \frac{ep}{1+e}$ $\quad -12 = \frac{ep}{1-e}$

$ep = 4+4e = -12+12e$

$16 = 8e$

$e = 2$

$ep = 4+4(2)$

$2p = 12$

$p = 6$

$r = \frac{12}{1+2\sin\theta}$

☺ BONUS QUESTIONS ☺

Convert the polar equation $r = \cos 4\theta$ to rectangular form and **SIMPLIFY**. **SCORE: ___ / 6 POINTS**

Show algebraically that the graph of polar equation $r = \pi - \theta$ is symmetric over $\theta = \frac{\pi}{2}$. **SCORE: ___ / 4 POINTS**