Math 49B (8:30am – 9:20am)
Midterm 3
Wed Nov 26, 2008

SCORE: ___ / 120 POINTS

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO MATRIX-CAPABLE OR GRAPHING CALCULATORS ALLOWED ON THIS MIDTERM

Find the magnitude and direction angle of $\mathbf{u} = -7\mathbf{i} - 7\mathbf{j}$. **SCORE: ___ / 4 POINTS**

$$\|\vec{u}\| = \sqrt{(-7)^2+(-7)^2} = 7\sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{-7}{-7}\right)+\pi = \frac{\pi}{4}+\pi = \frac{5\pi}{4}$$

Determine if the determinant equation $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} kc-a & kd-b \\ c & d \end{vmatrix}$ is true or false. **SCORE: ___ / 8 POINTS**

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc \qquad \begin{vmatrix} kc-a & kd-b \\ c & d \end{vmatrix} = (kc-a)d-(kd-b)c$$

$$= kcd-ad-kdc+bc$$

$$= -ad+bc$$

FALSE

Vector $\mathbf{u}$ has magnitude 4 and direction angle $\frac{4\pi}{3}$. Vector $\mathbf{v}$ has magnitude 6 and direction angle $\frac{5\pi}{6}$. **SCORE: ___ / 8 POINTS**

Find the component form of $\mathbf{u} + \mathbf{v}$.

$$\vec{u} = \left\langle 4\cos\tfrac{4\pi}{3}, 4\sin\tfrac{4\pi}{3}\right\rangle = \left\langle 4\left(-\tfrac{1}{2}\right), 4\left(-\tfrac{\sqrt{3}}{2}\right)\right\rangle = \langle -2, -2\sqrt{3}\rangle$$

$$\vec{v} = \left\langle 6\cos\tfrac{5\pi}{6}, 6\sin\tfrac{5\pi}{6}\right\rangle = \left\langle 6\left(-\tfrac{\sqrt{3}}{2}\right), 6\left(\tfrac{1}{2}\right)\right\rangle = \langle -3\sqrt{3}, 3\rangle$$

$$\vec{u}+\vec{v} = \langle -2-3\sqrt{3}, 3-2\sqrt{3}\rangle$$

Find the angle between the vectors $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{v} = -\mathbf{i} + 5\mathbf{j}$. **SCORE: ___ / 6 POINTS**

$$\theta = \cos^{-1}\frac{\langle 3,-2\rangle\cdot\langle -1,5\rangle}{\|\langle 3,-2\rangle\|\,\|\langle -1,5\rangle\|}$$

$$= \cos^{-1}\frac{-3-10}{\sqrt{3^2+(-2)^2}\,\sqrt{(-1)^2+5^2}}$$

$$= \cos^{-1}\frac{-13}{\sqrt{13}\,\sqrt{26}}$$

$$= \cos^{-1}\frac{-1}{\sqrt{2}}$$

$$= \frac{3\pi}{4} \text{ OR } 135^\circ$$

Morgan's retirement account has decreased in value to $40,000. She decides to switch some (but not all) of the remaining balance into 2 bonds, one earning 3% interest annually, and one earning 4% interest annually. She would like to earn a minimum of $1,000 interest annually, and she would like to invest at least twice as much in the 4% bond as in the 3% bond. Write, **but do not solve**, a system of inequalities to determine how Morgan can meet her goal. **State clearly what each unknown represents.** **SCORE: ___ / 10 POINTS**

DO NOT SOLVE YOUR SYSTEM OF INEQUALITIES

t = AMOUNT IN 3% BOND $\quad t \geq 0$

f = AMOUNT IN 4% BOND $\quad f \geq 0$

$t + f \leq 40{,}000$

$0.03t + 0.04f \geq 1{,}000$

$f \geq 2t$

A point lies 3 units below the **xy**-plane, 2 units right of the **xz**-plane and 4 units behind the **yz**-plane. **SCORE: ___ / 4 POINTS**

$-3 \quad z \quad 2 \quad y \quad -4 \quad x$

[a] What are the co-ordinates of the point ? $(-4, 2, -3)$

[b] In which octant does the point lie ? QUADRANT $2 + 4 =$ OCTANT 6

A line segment has endpoints (3, –2, 4) and (–2, –2, 1). **SCORE: ___ / 4 POINTS**

[a] Find the midpoint of the segment.

$$\left(\frac{3-2}{2}, \frac{-2-2}{2}, \frac{4+1}{2}\right) = \left(\frac{1}{2}, -2, \frac{5}{2}\right)$$

[b] Find the length of the segment.

$$\sqrt{(3-(-2))^2 + (-2-(-2))^2 + (4-1)^2} = \sqrt{5^2 + 0^2 + 3^2} = \sqrt{34}$$

Find the partial fraction decomposition of $\dfrac{2x^3 - 3x^2 - 3}{x^4 - 1}$. **SCORE: ___ / 12 POINTS**

$= \dfrac{2}{x+1} - \dfrac{1}{x-1} + \dfrac{x}{x^2+1}$

$$\frac{2x^3-3x^2-3}{(x^2-1)(x^2+1)} = \frac{2x^3-3x^2-3}{(x+1)(x-1)(x^2+1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$$

$$2x^3 - 3x^2 - 3 = A(x-1)(x^2+1) + B(x+1)(x^2+1) + Cx(x+1)(x-1) + D(x+1)(x-1)$$

$x = 1 \quad -4 = B(2)(2) \Rightarrow B = -1$

$x = -1 \quad -8 = A(-2)(2) \Rightarrow A = 2$

$x = 0 \quad -3 = A(-1)(1) + B(1)(1) + D(1)(-1)$

$-3 = -2 - 1 - D \Rightarrow D = 0$

COEF OF x^3: $\quad 2 = A + B + C$

$2 = 2 - 1 + C \Rightarrow C = 1$

Determine if the vectors <2, –6, –8> and <–1, 3, –4> are perpendicular, parallel or neither. **SCORE: ___ / 8 POINTS**

$\langle 2,-6,-8\rangle \cdot \langle -1,3,-4\rangle = -2-18+32 \neq 0$ NOT $\perp$

$\langle 2,-6,-8\rangle = k\langle 1,3,-4\rangle$

$2 = -k \Rightarrow k=-2$
$-6 = 3k$
$-8 = -4k \Rightarrow k=2$

} IMPOSSIBLE NOT $\parallel$

If **u** = <3, 2> and **v** = <1, –1>, write **u** as the sum of two vectors, one parallel to **v**, and one orthogonal to **v**. **SCORE: ___ / 10 POINTS**

$\text{PROJ}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{v} = \frac{3-2}{1+1}\langle 1,-1\rangle = \frac{1}{2}\langle 1,-1\rangle = \langle \frac{1}{2}, -\frac{1}{2}\rangle$

$\vec{b} = \vec{u} - \text{PROJ}_{\vec{v}}\vec{u} = \langle 3,2\rangle - \langle \frac{1}{2}, -\frac{1}{2}\rangle = \langle \frac{5}{2}, \frac{5}{2}\rangle$

$\vec{u} = \langle \frac{1}{2}, -\frac{1}{2}\rangle + \langle \frac{5}{2}, \frac{5}{2}\rangle$

$\parallel$ $\perp$

Using determinants, determine if the matrix $\begin{bmatrix} -4 & 0 & -2 & 7 \\ 2 & 4 & -1 & -3 \\ -8 & 0 & -4 & 1 \\ 6 & 0 & 3 & 0 \end{bmatrix}$ has an inverse. **SCORE: ___ / 10 POINTS**

$$4\begin{vmatrix} -4 & -2 & 7 \\ -8 & -4 & 1 \\ 6 & 3 & 0 \end{vmatrix}\begin{matrix} -4 & -2 \\ -8 & -4 \\ 6 & 3 \end{matrix}$$

$= 4(0 - 12 - 168 - -168 - -12 - 0)$

$= 0$

NO INVERSE

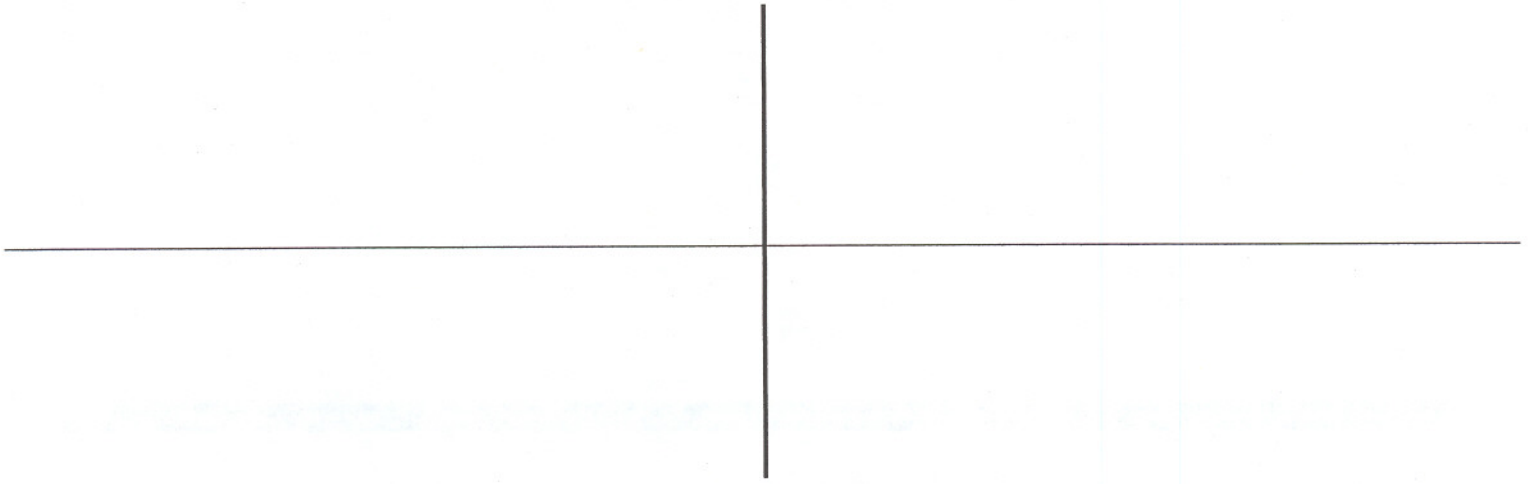
Vector **u** has initial point $(-3, 1, -4)$ and terminal point $(-6, -1, 2)$. Vector **v** has magnitude 8 and is in the same direction as **u**. Find **v**. **SCORE: ___ / 6 POINTS**

$\vec{u} = \langle -6 - -3, -1 - 1, 2 - -4 \rangle = \langle -3, -2, 6 \rangle$

$\frac{1}{\|\vec{u}\|}\vec{u} = \frac{1}{\sqrt{9+4+36}}\langle -3, -2, 6 \rangle = \frac{1}{7}\langle -3, -2, 6 \rangle$

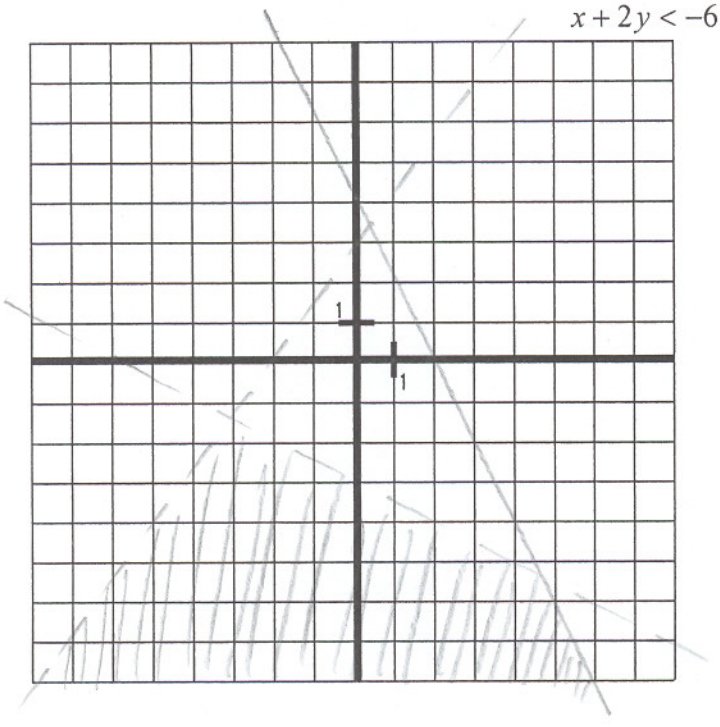
$\vec{v} = \frac{8}{7}\langle -3, -2, 6 \rangle = \langle \frac{-24}{7}, \frac{-16}{7}, \frac{48}{7} \rangle$

The vectors **u** and **v** are shown in the diagram below. Sketch (and clearly label) the vectors $2\mathbf{v} + 3\mathbf{u}$ and $3\mathbf{u} - \mathbf{v}$. **SCORE: ___ / 6 POINTS**



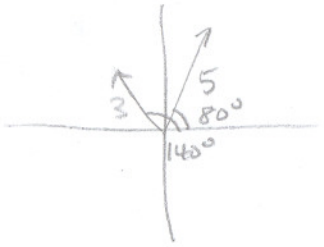
Graph the solution of the system of inequalities $3x - 2y > -6$, $2x + y \leq 4$, $x + 2y < -6$. **SCORE: ___ / 10 POINTS**

	X-INT	Y-INT	(0,0)?
$3x - 2y > -6$	-2	3	✓
$2x + y \leq 4$	2	4	✓
$x + 2y < -6$	-6	-3	✗



Vector **u** has magnitude 3 and direction angle $140°$. Vector **v** has magnitude 5 and direction angle $80°$. Find **u** · **v**. **HINT: Sketch the vectors first.**

SCORE: ___ / 8 POINTS



$\theta = 60°$

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$

$= 3 \cdot 5 \cdot \cos 60°$

$= \frac{15}{2}$

Find the center and radius of the sphere $x^2 + y^2 + z^2 - 2x + 6y - 10z - 1 = 0$.

SCORE: ___ / 6 POINTS

$x^2 - 2x + y^2 + 6y + z^2 - 10z = 1$

$(x^2 - 2x + 1) + (y^2 + 6y + 9) + (z^2 - 10z + 25) = 1 + 1 + 9 + 25$

$(x-1)^2 + (y+3)^2 + (z-5)^2 = 36$

CENTER $(1, -3, 5)$ RADIUS 6

☺ BONUS QUESTIONS ☺

Find a 3×3 matrix A with at least 7 non-zero entries such that $|A| = 8$.

SCORE: ___ / 5 POINTS

Find a vector **u** with integer components such that the angle between **u** and $4\mathbf{i} + \mathbf{j}$ is $45°$.

SCORE: ___ / 5 POINTS

HINT: This can be solved using a technique from Math 49A.