Math 1B
Group Quiz 1
Wed Apr 16, 2008

Names: ________________

[10 POINTS] Find the area under the curve $f(x)=2x^2-3x$ from $x=3$ to $x=5$ using the limit of the right handed sums.

$$\Delta x = \frac{5-3}{n} = \frac{2}{n}$$

$$x_i^* = 3 + i\left(\frac{2}{n}\right) = 3+\frac{2i}{n}$$

1

$$\sum_{i=1}^{n} f(x_i^*)\Delta x = \sum_{i=1}^{n}\left(2\left(3+\frac{2i}{n}\right)^2 - 3\left(3+\frac{2i}{n}\right)\right)\frac{2}{n}$$

1 1

$$= \frac{2}{n}\sum_{i=1}^{n}\left(2\left(9+\frac{12i}{n}+\frac{4i^2}{n^2}\right)-\left(9+\frac{6i}{n}\right)\right)$$

$$= \frac{2}{n}\sum_{i=1}^{n}\left(18+\frac{24i}{n}+\frac{8i^2}{n^2}-9-\frac{6i}{n}\right)$$

1

$$= \frac{2}{n}\sum_{i=1}^{n}\left(9+\frac{18i}{n}+\frac{8i^2}{n^2}\right)$$

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$$= \frac{2}{n}\left(9n + \frac{\cancel{18}^{\,9}}{\cancel{n}}\,\frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{\cancel{8}^{\,4}}{n^2}\,\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}_{\,3}}\right)$$

$$= \frac{2}{n}\left(9n+9(n+1)+\frac{4}{3n}\left(2n^2+3n+1\right)\right)\cdot\frac{3n}{3n}$$

$$= \frac{2}{3n^2}\left(27n^2+27n^2+27n+8n^2+12n+4\right)$$

2

$$= \frac{2}{3n^2}\left(62n^2+39n+4\right)$$

1

$$\lim_{n\to\infty}\frac{2}{3n^2}\left(62n^2+39n+4\right) = \frac{2\cdot 62}{3} = \frac{124}{3} = 41\tfrac{1}{3}$$

EITHER ONE IS OK

SANITY CHECK $n=1,\ \Delta x = 2$

$$f(5) = 2(5)^2 - 3(5) = 50-15 = 35$$

$$35\times 2 = 70$$

$$\frac{2}{3}(62+39+4) = \frac{2}{\cancel{3}}\cancel{105}^{\,35} = 70$$

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