Math 1B
Group Quiz 1
Wed Apr 16, 2008

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[10 POINTS] Find the area under the curve $f(x) = 3x^2 - 2x$ from $x = 2$ to $x = 5$ using the limit of the right handed sums.

$$\Delta x = \frac{5-2}{n} = \frac{3}{n}$$

$$x_i^* = 2 + i\left(\frac{3}{n}\right) = 2 + \frac{3i}{n} \quad 1$$

$$\sum_{i=1}^{n} f(x_i^*)\Delta x = \sum_{i=1}^{n} \left(3\left(2+\frac{3i}{n}\right)^2 - 2\left(2+\frac{3i}{n}\right)\right)\frac{3}{n} \quad 1 \quad 1$$

$$= \frac{3}{n}\sum_{i=1}^{n}\left(3\left(4+\frac{12i}{n}+\frac{9i^2}{n^2}\right) - \left(4+\frac{6i}{n}\right)\right)$$

$$= \frac{3}{n}\sum_{i=1}^{n}\left(12+\frac{36i}{n}+\frac{27i^2}{n^2} - 4 - \frac{6i}{n}\right)$$

$$= \frac{3}{n}\sum_{i=1}^{n}\left(8+\frac{30i}{n}+\frac{27i^2}{n^2}\right) \quad 1$$

$$= \frac{3}{n}\left(8n \overset{1}{} + \frac{\cancel{30}^{15}}{\cancel{n}}\frac{\cancel{n}(n+1)}{\cancel{2}} \overset{1}{} + \frac{\cancel{27}^{9}}{n^2}\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}_{2}}\right) \quad 1$$

$$= \frac{3}{n}\left(8n + 15(n+1) + \frac{9}{2n}(2n^2+3n+1)\right)\cdot\frac{2n}{2n}$$

$$= \frac{3}{2n^2}\left(16n^2 + 30n^2 + 30n + 18n^2 + 27n + 9\right)$$

$$= \frac{3}{2n^2}\left(64n^2 + 57n + 9\right) \quad 2$$

$$\lim_{n\to\infty} \frac{3}{2n^2}\left(64n^2 + 57n + 9\right) = \frac{3\cdot\cancel{64}^{32}}{\cancel{2}} = 96 \quad 1$$

SANITY CHECK $n = 1$, $\Delta x = 3$

$$f(5) = 3(5)^2 - 2(5) = 75 - 10 = 65$$

$$65 \times 3 = 195$$

$$\frac{3}{2}(64+57+9) = \frac{3}{\cancel{2}}\cancel{130}^{65} = 195 \quad ✓$$