Math 1B
Group Quiz 3
Wed May 7, 2008

SUBTRACT $\frac{1}{4}$ POINT EACH TIME YOU FORGET du, dx OR $+C$

Names: _____

THIS IS A NO CALCULATOR QUIZ

[2 POINTS EACH] Evaluate the following integrals.

SHOW ALL WORK
DO NOT USE FORMULAE FROM THE TABLE OF INTEGRALS

$$\int \frac{4}{x^{1/4}\left(x^{3/4}+1\right)}\,dx$$

$u = x^{\frac{3}{4}} + 1$

$\frac{du}{dx} = \frac{3}{4}x^{-\frac{1}{4}}$

$\frac{4}{3}x^{\frac{1}{4}}\,du = dx$

$\frac{4}{x^{\frac{1}{4}}(x^{\frac{3}{4}}+1)}\,\frac{4}{3}x^{\frac{1}{4}}\,du$

$\frac{16}{9}\frac{1}{u}\,du$

$= \int \frac{16}{9}\frac{1}{u}\,du$ (1)

$= \frac{16}{9}\ln|u| + C$ ($\frac{1}{2}$)

$= \frac{16}{9}\ln\left|x^{\frac{3}{4}}+1\right| + C$ ($\frac{1}{2}$)

$= \frac{16}{9}\ln\left(x^{\frac{3}{4}}+1\right) + C$ ($\frac{1}{2}$)

$$\int \frac{x^2}{x^6+1}\,dx$$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$\frac{1}{3x^2}\,du = dx$

$\frac{x^2}{x^6+1}\,\frac{1}{3x^2}\,du$

$\frac{1}{3}\frac{1}{u^2+1}\,du$

$= \int \frac{1}{3}\frac{1}{u^2+1}\,du$ (1)

$= \frac{1}{3}\tan^{-1}u + C$ ($\frac{1}{2}$)

$= \frac{1}{3}\tan^{-1}x^3 + C$ ($\frac{1}{2}$)

$$\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x}\,dx$$

$= \int_0^{\frac{\pi}{3}} \sec x \tan x\,dx$ ($\frac{1}{2}$)

$= \sec x \Big|_0^{\frac{\pi}{3}}$ ($\frac{1}{2}$)

$= \sec\frac{\pi}{3} - \sec 0$

$= 2 - 1$

$= 1$ ($\frac{1}{2}$)

$$\int_{-1}^{1} e^{\ln(1+x^4)}\,dx$$

$= \int_{-1}^{1} (1+x^4)\,dx$ ($\frac{1}{2}$)

$= x + \frac{1}{5}x^5 \Big|_{-1}^{1}$ ($\frac{1}{2}$)

$= \left(1+\frac{1}{5}\right) - \left(-1-\frac{1}{5}\right)$

$= \frac{12}{5}$ ($\frac{1}{2}$)

OR

$$\int \frac{1}{\sqrt{4-x^2}}\,dx$$

$u = \frac{x}{2}$

$\frac{du}{dx} = \frac{1}{2}$

$2\,du = dx$

$\frac{1}{2\sqrt{1-\frac{x^2}{4}}}\,2\,du$

$\frac{1}{\sqrt{1-u^2}}\,du$

$= \int \frac{1}{2\sqrt{1-\frac{x^2}{4}}}\,dx$ ($\frac{1}{2}$)

$= \int \frac{1}{\sqrt{1-u^2}}\,du$ (1)

$= \sin^{-1}u + C$ ($\frac{1}{2}$)

$= \sin^{-1}\frac{x}{2} + C$ ($\frac{1}{2}$)

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$-\frac{1}{\sin x}\,du = dx$

$\frac{\sin x}{\cos^2 x}\cdot -\frac{1}{\sin x}\,du$

$-\frac{1}{u^2}\,du$

$\int_1^{\frac{1}{2}} -\frac{1}{u^2}\,du$ ($\frac{1}{2}$)

$= \frac{1}{u}\Big|_1^{\frac{1}{2}}$ ($\frac{1}{2}$)

$= 2 - 1$

$= 1$ ($\frac{1}{2}$)