Math 1B
Group Quiz 4
Tue May 13, 2008

READ THIS

Names: ______________________

This was an odd quiz. Three of the four questions had more than 1 possible solution each. So, for each question, make sure you compare your work against the solution that it most closely resembles.

The questions are being presented out of order, so make sure you read the questions carefully.

$\int \sin 2x \cos 4x \, dx$ **(3 SOLUTIONS)**

$\frac{7}{4}$ POINTS TOTAL

u	dv
$\sin 2x$	$\cos 4x$
$2\cos 2x$	$\frac{1}{4}\sin 4x$
$-4\sin 4x$	$-\frac{1}{16}\cos 4x$

SOLUTION 1:

$$\int \sin 2x \cos 4x \, dx = \boxed{\frac{1}{4}\sin 2x \sin 4x} + \boxed{\frac{1}{8}\cos 2x \cos 4x} + \boxed{\frac{1}{4}\int \sin 2x \cos 4x \, dx}$$

$$\boxed{\frac{3}{4}\int \sin 2x \cos 4x \, dx} = \frac{1}{4}\sin 2x \sin 4x + \frac{1}{8}\cos 2x \cos 4x$$

$$\int \sin 2x \cos 4x \, dx = \boxed{\frac{1}{3}\sin 2x \sin 4x} + \boxed{\frac{1}{6}\cos 2x \cos 4x} + \boxed{C}$$

$\frac{1}{4}$ POINT EACH

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u	dv
$\cos 4x$	$\sin 2x$
$-4\sin 4x$	$-\frac{1}{2}\cos 2x$
$-16\cos 4x$	$-\frac{1}{4}\sin 2x$

SOLUTION 2:

$$\int \sin 2x \cos 4x \, dx = -\frac{1}{2}\cos 4x \cos 2x - \sin 4x \sin 2x + \frac{1}{4}\int \sin 2x \cos 4x \, dx$$

$$\frac{3}{4}\int \sin 2x \cos 4x \, dx = -\frac{1}{2}\cos 4x \cos 2x - \sin 4x \sin 2x$$

$$\int \sin 2x \cos 4x \, dx = -\frac{2}{3}\cos 4x \cos 2x - \frac{4}{3}\sin 4x \sin 2x + C$$

$\frac{1}{4}$ POINT EACH

SOLUTION 3:

$$\int \sin 2x \cos 4x \, dx$$

$$= \int \frac{1}{2}(\sin 6x + \sin 2x)\, dx \qquad \frac{3}{4}$$

$$= \frac{1}{2}\left(-\frac{1}{6}\cos 6x - \frac{1}{2}\cos 2x\right) + C \qquad \frac{1}{2}$$

$$= -\frac{1}{12}(\cos 6x + 3\cos 2x) + C$$

$\frac{1}{4}$ $\frac{1}{4}$

FINAL ANSWER MUST EITHER BE COMPLETELY FACTORED AS SHOWN, OR COMPLETELY DISTRIBUTED

$\int \sin^2 x \cos^2 x\, dx$ **(3 SOLUTIONS)**

$\frac{11}{4}$ POINTS TOTAL

$$\int \sin^2 x \cos^2 x\, dx$$

$$= \int \sin^2 x\,(1-\sin^2 x)\, dx$$

$$= \int (\sin^2 x - \sin^4 x)\, dx$$

$$= \int \sin^2 x\, dx - \int \sin^4 x\, dx \quad \frac{1}{2}$$

SOLUTION 1:

$$= \int \sin^2 x\, dx - \left(-\frac{1}{4}\sin^3 x\cos x + \frac{3}{4}\int \sin^2 x\, dx\right) \quad \frac{1}{2}$$

$$= \frac{1}{4}\sin^3 x\cos x + \frac{1}{4}\int \sin^2 x\, dx$$

$$= \frac{1}{4}\sin^3 x\cos x + \frac{1}{4}\left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right) + C$$

$$= \frac{1}{4}\sin^3 x\cos x + \frac{1}{8}x - \frac{1}{16}\sin 2x + C \quad \frac{1}{4} \quad OR \quad \frac{1}{4}\sin^3 x\cos x - \frac{1}{8}\sin x\cos x + \frac{1}{8}x + C$$

$1\frac{1}{2}$ FOR EITHER ONE

$$\int \sin^2 x \cos^2 x\, dx$$

$$= \int (1-\cos^2 x)\cos^2 x\, dx$$

$$= \int (\cos^2 x - \cos^4 x)\, dx$$

$$= \int \cos^2 x\, dx - \int \cos^4 x\, dx \quad \frac{1}{2}$$

SOLUTION 2:

$$= \int \cos^2 x\, dx - \left(\frac{1}{4}\cos^3 x\sin x + \frac{3}{4}\int \cos^2 x\, dx\right) \quad \frac{1}{2}$$

$$= -\frac{1}{4}\cos^3 x\sin x + \frac{1}{4}\int \cos^2 x\, dx$$

$$= -\frac{1}{4}\cos^3 x\sin x + \frac{1}{4}\left(\frac{1}{2}x + \frac{1}{4}\sin 2x\right) + C$$

$$= -\frac{1}{4}\cos^3 x\sin x + \frac{1}{8}x + \frac{1}{16}\sin 2x + C \quad \frac{1}{4} \quad OR \quad -\frac{1}{4}\cos^3 x\sin x + \frac{1}{8}\cos x\sin x + \frac{1}{8}x + C$$

$1\frac{1}{2}$ FOR EITHER ONE

SOLUTION 3:

$$\int \sin^2 x \cos^2 x \, dx$$

$$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx \quad \frac{1}{2}$$

$$= \frac{1}{4}\int \left(1-\cos^2 2x\right) dx \quad \frac{1}{2}$$

$$= \frac{1}{4}\int \left(1-\left(\frac{1+\cos 4x}{2}\right)\right) dx \quad \frac{1}{2}$$

$$= \frac{1}{8}\int \left(2-\left(1+\cos 4x\right)\right) dx$$

$$= \frac{1}{8}\int \left(1-\cos 4x\right) dx \quad \frac{1}{4}$$

$$= \frac{1}{8}\left(x-\frac{1}{4}\sin 4x\right)+C \quad \frac{1}{2}$$

$$= \frac{1}{32}\left(4x-\sin 4x\right)+C \quad \frac{1}{4}$$

$$\frac{1}{4}$$

FINAL ANSWER MUST BE EITHER COMPLETELY FACTORED AS SHOWN, OR COMPLETELY DISTRIBUTED

$\int \sin^3 x \cos^7 x \, dx$ **(2 SOLUTIONS)**

$\frac{11}{4}$ POINTS TOTAL

$$\int \sin^3 x \cos^7 x \, dx$$

$$= \int \sin^2 x \cos^7 x \sin x \, dx$$

$$= \int (1-\cos^2 x) \cos^7 x \sin x \, dx$$

SOLUTION 1:

$\frac{1}{2}$ $$= -\int (1-u^2)u^7 \, du \qquad \text{if } u = \cos x \text{ and } du = -\sin x \, dx$$ $\frac{1}{2}$

$\frac{1}{2}$ $$= -\int (u^7 - u^9) \, du$$

$\frac{1}{2}$ $$= -\frac{1}{8}u^8 + \frac{1}{10}u^{10} + C$$

$\frac{1}{2}$ $$= -\frac{1}{8}\cos^8 x + \frac{1}{10}\cos^{10} x + C$$ $\frac{1}{4}$

$$\int \sin^3 x \cos^7 x \, dx$$

$$= \int \sin^3 x \cos^6 x \cos x \, dx$$

$$= \int \sin^3 x (1-\sin^2 x)^3 \cos x \, dx$$

$\frac{1}{2}$ $$= \int u^3 (1-u^2)^3 \, du \qquad \text{if } u = \sin x \text{ and } du = \cos x \, dx$$ $\frac{1}{2}$

SOLUTION 2:

$$= \int u^3 (1 - 3u^2 + 3u^4 - u^6) \, du$$

$\frac{1}{2}$ $$= \int (u^3 - 3u^5 + 3u^7 - u^9) \, du$$

$\frac{1}{2}$ $$= \frac{1}{4}u^4 - \frac{1}{2}u^6 + \frac{3}{8}u^8 - \frac{1}{10}u^{10} + C$$

$\frac{1}{2}$ $$= \frac{1}{4}\sin^4 x - \frac{1}{2}\sin^6 x + \frac{3}{8}\sin^8 x - \frac{1}{10}\sin^{10} x + C$$ $\frac{1}{4}$

THIS QUESTION COULD BE DONE USING $\sin^4 x = (1-\cos^2 x)^2$ AND REDUCTION FORMULAE BUT THAT METHOD IS SO INEFFICIENT THAT IT MEANS YOU PROBABLY DON'T KNOW HOW TO HANDLE THIS PROPERLY, SO YOU SHOULD RECEIVE 0 POINTS ANYWAY

$\int \sin^4 x \cos^5 x \, dx$ **(1 SOLUTION)**

$\frac{11}{4}$ POINTS TOTAL

$$= \int \sin^4 x \cos^4 x \cos x \, dx$$

$$= \int \sin^4 x (1-\sin^2 x)^2 \cos x \, dx$$

$\frac{1}{2}$ $$= \int u^4 (1-u^2)^2 \, du \qquad \text{if } u = \sin x \text{ and } du = \cos x \, dx$$ $\frac{1}{2}$

$$= \int u^4 (1 - 2u^2 + u^4) \, du$$

$\frac{1}{2}$ $$= \int (u^4 - 2u^6 + u^8) \, du$$

$\frac{1}{2}$ $$= \frac{1}{5}u^5 - \frac{2}{7}u^7 + \frac{1}{9}u^9 + C$$

$\frac{1}{2}$ $$= \frac{1}{5}\sin^5 x - \frac{2}{7}\sin^7 x + \frac{1}{9}\sin^9 x + C$$ $\frac{1}{4}$