

THIS IS A NO CALCULATOR QUIZ

[10 POINTS] Evaluate the following integrals.

SHOW ALL WORK

$$\int \frac{4}{x^3 + 2x^2 + 2x} dx = \int \frac{4}{x(x^2 + 2x + 2)} dx = \int \left(\frac{A}{x} + \frac{B(2x+2) + C}{x^2 + 2x + 2} \right) dx$$

$$4 = A(x^2 + 2x + 2) + B(2x + 2)x + Cx$$

$$x=0: 4 = 2A \Rightarrow A = 2$$

$$x=-1: 4 = A - C \Rightarrow C = A - 4 = -2$$

$$\text{COEF OF } x^2: 0 = A + 2B \Rightarrow B = -\frac{1}{2}A = -1$$

$$= \int \left(\frac{2}{x} - \frac{2x+2}{x^2+2x+2} - \frac{2}{(x+1)^2+1} \right) dx$$

$$u = x^2 + 2x + 2$$

$$du = (2x+2)dx$$

$$u = x+1$$

$$du = dx$$

$$= 2 \ln|x| - \ln(x^2 + 2x + 2) - 2 \tan^{-1}(x+1) + C$$

$\frac{1}{4}$

$\frac{1}{2}$

$\frac{1}{4}$

$$\int \frac{x^4 + 2}{x^3 + 2x^2 + x} dx$$

$$\begin{array}{r} x^3 + 2x^2 + x \overline{) \frac{x-2}{x^4}} \\ \underline{x^4 + 2x^3 + x^2} \\ -2x^3 - x^2 \\ \underline{-2x^3 - 4x^2 - 2x} \\ 3x^2 + 2x + 2 \end{array}$$

$$= \int \left(\frac{x-2}{1} + \frac{3x^2+2x+2}{x(x+1)^2} \right) dx$$

$$= \frac{1}{2}x^2 - 2x + \int \left(\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \right) dx$$

$$3x^2 + 2x + 2 = A(x+1)^2 + Bx(x+1) + Cx$$

$$x=0: 2 = A$$

$$x=-1: 3 = -C \Rightarrow C = -3$$

$$\text{COEF OF } x^2: 3 = A + B \Rightarrow B = 3 - A = 1$$

$$= \frac{1}{2}x^2 - 2x + \int \left(\frac{2}{x} + \frac{1}{x+1} - \frac{3}{(x+1)^2} \right) dx$$

$$= \frac{1}{2}x^2 - 2x + 2\ln|x| + \ln|x+1| + \frac{3}{x+1} + C$$

$\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{4}$

$u = x+1$
 $du = dx$