Math 1B
Group Quiz 7
Wed Jun 4, 2008

Names: ______________________
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THIS IS A NO CALCULATOR QUIZ

[3 POINTS] Find the area between the curves $y = x^2 + 4$ and $x = 2 - 2y$ on the interval $[0,2]$.

$$y = \frac{2-x}{2} = 1 - \frac{1}{2}x$$

$\frac{1}{2}$ $\frac{1}{2}$

$$\int_0^2 \left(x^2+4 - \left(1-\tfrac{1}{2}x\right)\right) dx$$

$$= \int_0^2 \left(x^2 + \tfrac{1}{2}x + 3\right) dx$$

$$= \left.\tfrac{1}{3}x^3 + \tfrac{1}{4}x^2 + 3x\right|_0^2$$

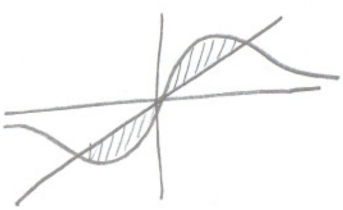
1

$$= \frac{8}{3} + 1 + 6$$

$$= \frac{29}{3}$$

1

[3 POINTS] Find the area of the region determined by the intersections of the curves $y = \dfrac{5x}{1+x^2}$ and $y = x$.



$$\frac{5x}{1+x^2} = x$$

$$5x = x + x^3$$

$$0 = x^3 - 4x$$

$$0 = x(x^2-4)$$

$$0 = x(x+2)(x-2)$$

$$x = 0, 2, -2$$

1

$\frac{1}{2}$

$$2\int_0^2 \left(\frac{5x}{1+x^2} - x\right) dx$$

$$= 2\left(\left.\tfrac{5}{2}\ln(1+x^2) - \tfrac{1}{2}x^2\right)\right|_0^2$$

1

$$= 2\left(\tfrac{5}{2}\ln 5 - 2\right)$$

$$= 5\ln 5 - 4$$

$\frac{1}{2}$

OR

$\frac{1}{2}$

$$\int_{-2}^0 \left(x - \frac{5x}{1+x^2}\right) dx + \int_0^2 \left(\frac{5x}{1+x^2} - x\right) dx$$

1

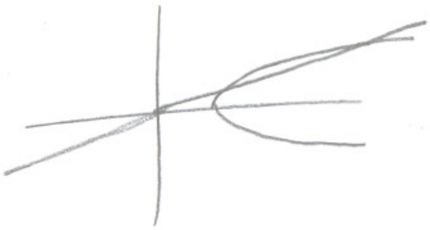
$$= \left.\left(\tfrac{1}{2}x^2 - \tfrac{5}{2}\ln(1+x^2)\right)\right|_{-2}^0 + \left.\left(\tfrac{5}{2}\ln(1+x^2) - \tfrac{1}{2}x^2\right)\right|_0^2$$

$$= -\left(2 - \tfrac{5}{2}\ln 5\right) + \tfrac{5}{2}\ln 5 - 2 = 5\ln 5 - 4$$

$\frac{1}{2}$

QUESTIONS ON OTHER SIDE

[4 POINTS] Find the area of the region bounded by the curves $x = 5y$ and $x = 6 + y^2$. $\leftarrow y = \frac{1}{5}x, \ y = \sqrt{x-6}$



$5y = 6 + y^2$

$0 = y^2 - 5y + 6$

$0 = (y-2)(y-3)$

$y = 2, 3$ 1

$x = 10, 15$ 1

$$\int_2^3 (5y - (6 + y^2))\,dy \quad 1$$

$$= \int_2^3 (-y^2 + 5y - 6)\,dy$$

$$= -\frac{1}{3}y^3 + \frac{5}{2}y^2 - 6y \Big|_2^3 \quad 1$$

$$= -\frac{1}{3}(27-8) + \frac{5}{2}(9-4) - 6(3-2)$$

$$= -\frac{19}{3} + \frac{25}{2} - 6 = \frac{-38 + 75 - 36}{6} = \frac{1}{6} \quad 1$$

OR

$$1 \quad \int_{10}^{15} \left(\sqrt{x-6} - \frac{1}{5}x\right) dx$$

$$= 1 \quad \frac{2}{3}(x-6)^{\frac{3}{2}} - \frac{1}{10}x^2 \Big|_{10}^{15}$$

$$= \frac{2}{3}\left(9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right) - \frac{1}{10}\left(15^2 - 10^2\right)$$

$$= \frac{2}{3}(27-8) - \frac{1}{10}(225-100)$$

$$= \frac{38}{3} - \frac{25}{2} = \frac{76-75}{6}$$

$$= \frac{1}{6} \quad 1$$

[4 BONUS POINTS] Consider two parabolas, each of which has its vertex at $x = 0$, but with different concavities. Let h be the difference in the y-coordinates of the vertices, and let w be the difference in the x-coordinates of the intersection points. Show that the area between the curves is $\frac{2}{3}hw$.