Math 1B
Midterm 1
Wed Apr 30, 2008

What day of the month is your birthday ? __ __
What are the last 2 digits of your address ? __ __
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CALCULATORS ARE NOT ALLOWED ON THIS SECTION

[10 POINTS] Write both parts of the Fundamental Theorem of Calculus. **DO NOT INCLUDE THE *NET CHANGE THEOREM*.**

① IF f IS CONTINUOUS ON $[a,b]$
AND F IS ANY ANTI-DERIVATIVE OF f,
THEN $\int_a^b f(x)\,dx = F(b) - F(a)$

② IF f IS CONTINUOUS AND a IS A CONSTANT,
AND $F(x) = \int_a^x f(t)\,dt$,
THEN $F'(x) = f(x)$

[10 POINTS] **MULTIPLE CHOICE (NO PARTIAL CREDIT)**

Estimate the area under the graph of f on the interval [2, 10] using 4 subintervals and midpoint evaluation.

x	0	1	2	3	4	5	6	7	8	9	10	11	12
$f(x)$	7	9	10	13	12	10	7	3	2	4	5	2	−1

[A] 63 [B] 58 [C] 60
[D] 66 [E] 62 [F] 65

LETTER OF CORRECT ANSWER: C

[10 POINTS] **MULTIPLE CHOICE (NO PARTIAL CREDIT)**

If f is continuous, decreasing and concave up on $[a, b]$, which of the following numerical integration methods give answers less than $\int_a^b f(x)\,dx$?

[A] left hand & midpoint [B] right hand & midpoint
[C] left hand & trapezoidal [D] right hand & trapezoidal
[E] left hand & right hand [F] midpoint & trapezoidal

LETTER OF CORRECT ANSWER: B

[10 POINTS] **MULTIPLE CHOICE (NO PARTIAL CREDIT)**

Estimate $\int_0^{12} f(x)\,dx$ using Simpson's Rule with $n = 6$.

x	0	1	2	3	4	5	6	7	8	9	10	11	12
$f(x)$	−1	0	2	1	−1	−1	1	3	2	0	1	2	4

[A] 12 [B] 14 [C] 16
[D] 17 [E] 19 [F] 20

LETTER OF CORRECT ANSWER: B

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THEN $F'(x) = f(x)$

[10 POINTS] **MULTIPLE CHOICE (NO PARTIAL CREDIT)**

Estimate the area under the graph of f on the interval [3, 11] using 4 subintervals and midpoint evaluation.

x	0	1	2	3	4	5	6	7	8	9	10	11	12
$f(x)$	7	9	10	13	12	10	7	3	2	4	5	2	–1

[A] 53 [B] 58 [C] 50
[D] 56 [E] 52 [F] 55

LETTER OF CORRECT ANSWER: E

[10 POINTS] **MULTIPLE CHOICE (NO PARTIAL CREDIT)**

If f is continuous, decreasing and concave down on [a, b], which of the following numerical integration methods give answers less than $\int_a^b f(x)\,dx$?

[A] left hand & midpoint
[B] right hand & midpoint
[C] left hand & trapezoidal
[D] right hand & trapezoidal
[E] left hand & right hand
[F] midpoint & trapezoidal

LETTER OF CORRECT ANSWER: D

[10 POINTS] **MULTIPLE CHOICE (NO PARTIAL CREDIT)**

Estimate $\int_0^{12} f(x)\,dx$ using Simpson's Rule with $n = 6$.

x	0	1	2	3	4	5	6	7	8	9	10	11	12
$f(x)$	–2	0	3	1	–1	–2	1	3	2	0	1	2	4

[A] 12 [B] 14 [C] 16
[D] 17 [E] 19 [F] 20

LETTER OF CORRECT ANSWER: C

[15 POINTS] Let $g(t)=\int_{\ln t}^{2t^2}\sqrt{x^3+1}\,dx$. Find $g'(1)$.

$$g'(t)=\sqrt{(2t^2)^3+1}\cdot 4t-\sqrt{(\ln t)^3+1}\cdot\frac{1}{t}$$

$$g'(1)=\sqrt{2^3+1}\cdot 4-\sqrt{0^3+1}\cdot 1$$

$$=\sqrt{9}\cdot 4-\sqrt{1}\cdot 1$$

$$=3\cdot 4-1\cdot 1$$

$$=12-1$$

$$=11$$

[20 POINTS] Compute the exact area under the graph of $f(x)=10-3x$ on the interval [1, 3] using the limit of the right-endpoint Riemann sum.

$$\Delta x=\frac{3-1}{n}=\frac{2}{n}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(10-3\left(1+\frac{2}{n}i\right)\right)\frac{2}{n}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left(10-3-\frac{6}{n}i\right)\frac{2}{n}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left(7-\frac{6}{n}i\right)$$

$$=\lim_{n\to\infty}\frac{2}{n}\left(7n-\frac{\cancel{6}^{\,3}}{\cancel{n}}\frac{\cancel{n}(n+1)}{\cancel{2}}\right)$$

$$=\lim_{n\to\infty}\frac{2}{n}(7n-3n-3)$$

$$=\lim_{n\to\infty}\frac{2}{n}(4n-3)$$

$$=2\cdot 4$$

$$=8$$

[45 POINTS] Evaluate the following integrals.

[a] $\int \frac{x^2}{2\sqrt{x+4}}\,dx$

$u = x+4 \qquad x = u-4$

$\frac{du}{dx} = 1$

$du = dx$

$\frac{x^2}{2\sqrt{x+4}}\,du = \frac{(u-4)^2}{2\sqrt{u}}\,du$

$= \frac{u^2-8u+16}{2u^{\frac{1}{2}}}\,du$

$= \left(\frac{1}{2}u^{\frac{3}{2}} - 4u^{\frac{1}{2}} + 8u^{-\frac{1}{2}}\right)du$

$\int \left(\frac{1}{2}u^{\frac{3}{2}} - 4u^{\frac{1}{2}} + 8u^{-\frac{1}{2}}\right)du$

$= \frac{1}{2}\cdot\frac{2}{5}u^{\frac{5}{2}} - 4\cdot\frac{2}{3}u^{\frac{3}{2}} + 8\cdot 2u^{\frac{1}{2}} + C$

$= \frac{1}{5}u^{\frac{5}{2}} - \frac{8}{3}u^{\frac{3}{2}} + 16u^{\frac{1}{2}} + C$

$= \frac{1}{5}(x+4)^{\frac{5}{2}} - \frac{8}{3}(x+4)^{\frac{3}{2}} + 16(x+4)^{\frac{1}{2}} + C$

[b] $\int_{\frac{\pi}{3}}^{0} \left(3\cos\frac{1}{2}x + \sin 2x - \sec x \tan x\right)dx$

$= 3\cdot 2\sin\frac{1}{2}x - \frac{1}{2}\cos 2x - \sec x \Big|_{\frac{\pi}{3}}^{0}$

$= 6\sin\frac{1}{2}x - \frac{1}{2}\cos 2x - \sec x \Big|_{\frac{\pi}{3}}^{0}$

$= \left(6\cdot 0 - \frac{1}{2}\cdot 1 - 1\right)$
$- \left(6\cdot\frac{1}{2} - \frac{1}{2}\left(-\frac{1}{2}\right) - 2\right)$

$= -\frac{1}{2} - 1 - 3 - \frac{1}{4} + 2$

$= -\frac{3}{4} - 2$

$= -\frac{11}{4}$

[c] $\int \left(e^{2x}-1\right)^2 dx$

$= \int \left(e^{4x} - 2e^{2x} + 1\right)dx$

$= \frac{1}{4}e^{4x} - e^{2x} + x + C$

[d] $\int_0^1 \frac{6+3x}{9-4x-x^2}\,dx$

$u = 9-4x-x^2$

$\frac{du}{dx} = -4-2x$

$\frac{-1}{2(2+x)}\,du = dx$

$\frac{3\cancel{(2+x)}}{9-4x-x^2}\cdot\frac{-1}{2\cancel{(2+x)}}\,du$

$= -\frac{3}{2}\frac{1}{u}\,du \qquad \begin{array}{l} x=0 \Rightarrow u=9 \\ x=1 \Rightarrow u=4 \end{array}$

$-\frac{3}{2}\int_9^4 \frac{1}{u}\,du = -\frac{3}{2}\ln|u|\Big|_9^4$

$= -\frac{3}{2}(\ln 4 - \ln 9)$

$= -\frac{3}{2}\ln\frac{4}{9}$ OR $\frac{3}{2}\ln\frac{9}{4}$

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YOU MAY USE YOUR CALCULATOR ON THIS SECTION

[20 POINTS] Consider the definite integral $\int_1^8 \frac{3}{\sqrt[3]{x}}\,dx$.

[a] What is the value of T_{20} ?

13.50950803

[b] What is the value of S_{12} ?

13.50137976

[c] Find the exact error in the approximation in [a].

$$\int_1^8 3x^{-\frac{1}{3}}\,dx$$
$$= 3\cdot\frac{3}{2}x^{\frac{2}{3}}\Big|_1^8$$
$$= \frac{9}{2}\left(8^{\frac{2}{3}} - 1^{\frac{2}{3}}\right)$$
$$= \frac{9}{2}(4-1)$$
$$= \frac{27}{2}$$

$$\frac{27}{2} - T_{20} = -0.0050803$$

[10 POINTS] Find bounds on the error when the Midpoint Rule with $n = 10$ is used to approximate $\int_1^4 e^{2x}\,dx$.

$$|EM_{10}| \le \frac{K(4-1)^3}{24\cdot 10^2}$$
$$= \frac{4e^8(27)}{2400}$$
$$= 134.143$$

$f = e^{2x}$
$f' = 2e^{2x}$
$f'' = 4e^{2x}$
$|4e^{2x}| \le 4e^{2(4)}$
$= 4e^8$

OR 295.245 IF YOU USED $e \approx 3$

BONUS POINTS ON OTHER SIDE