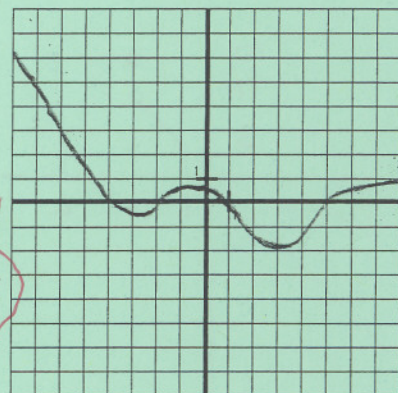


# THIS IS A NO CALCULATOR QUIZ

[6 POINTS] The graph of  $y = f(x)$  is shown to the right.

$y = f(x)$   
WINDOW  $[-8, 8] \times [-8, 8]$



- (a) Write the **total** area between the  $x$ -axis and  $y = f(x)$  as an integral or a sum/difference of integrals.

**DO NOT USE ABSOLUTE VALUES (ERIK'S TRICK)  
OR ROOTS OF POWERS (ANUJ'S TRICK).**

$$\int_{-8}^{-4} f(x) dx - \int_{-4}^{-2} f(x) dx + \int_{-2}^1 f(x) dx$$

$$- \int_1^5 f(x) dx + \int_5^8 f(x) dx$$

- (b) Determine whether  $\int_1^8 f(x) dx$  is positive or negative. **Explain briefly.**

2 **NEGATIVE** AREA UNDER  $x$ -AXIS ON  $[1, 5]$   
GREATER THAN  
AREA ABOVE  $x$ -AXIS ON  $[5, 8]$

[2 POINTS]

Write each expression below **as a single integral**.

(a)  $\int_8^5 f(x) dx + \int_3^8 f(x) dx$

$$\int_3^5 f(x) dx$$

(b)  $\int_{-3}^4 f(x) dx - \int_1^4 f(x) dx$

$$\int_{-3}^1 f(x) dx$$

[4 POINTS]

Write the definition of "definite integral":

THE DEFINITE INTEGRAL OF  $f$  ON  $[a, b]$  IS

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x \quad \text{WHERE } \Delta x = \frac{b-a}{n} \text{ AND } a + (i-1)\Delta x \leq x_i^* \leq a + i\Delta x$$

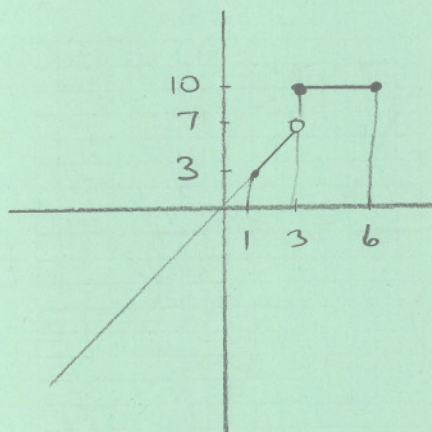
THE LIMIT MUST EXIST AND BE THE SAME REGARDLESS OF HOW THE  $x_i^*$  ARE CHOSEN.

SUBTRACT  $\frac{1}{2}$  POINT EACH TIME  
YOU DO N'T WRITE  $dx$



# THIS IS A NO CALCULATOR QUIZ

[4 POINTS] Let  $f(x) = \begin{cases} 2x+1 & \text{if } x < 3 \\ 10 & \text{if } x \geq 3 \end{cases}$ . Compute  $\int_1^6 f(x) dx$  using geometry. DO NOT USE ANTI-DERIVATIVES.



$$\begin{aligned}
 &= \frac{1}{2} (3+7) 2 + 10(3) \\
 &= 10 + 30 \\
 &= 40
 \end{aligned}$$

[2 POINTS] In each sentence below, circle the underlined word which completes the sentence correctly.

If  $f(x)$  is decreasing and concave down on  $[a, b]$ ,

- (a) a Riemann sum with midpoint evaluation points will be MORE / LESS than the area under  $f(x)$  on  $[a, b]$   
 (b) a Riemann sum with right-endpoint evaluation points will be MORE / LESS than the area under  $f(x)$  on  $[a, b]$

[2 POINTS] MULTIPLE CHOICE (NO PARTIAL CREDIT)

Evaluate the Riemann sum of  $f(x) = x^2 - 5x$  on  $[2, 10]$  using 4 subintervals, if the evaluation points are the midpoints of each subinterval.

[A] 88  
[D] 94

[B] 84  
[E] 86

[C] 90  
[F] 92

LETTER OF  
CORRECT ANSWER: A

[2 BONUS POINTS]

Sketch the graph of a continuous function  $f(x)$  such that the Riemann sum of  $f(x)$  on  $[a, b]$  using 2 subintervals is less than the area under  $f(x)$  on  $[a, b]$ , regardless of whether the evaluation points are left-endpoints, midpoints or right-endpoints. HINT: TRY FIRST WITH 1 SUBINTERVAL.