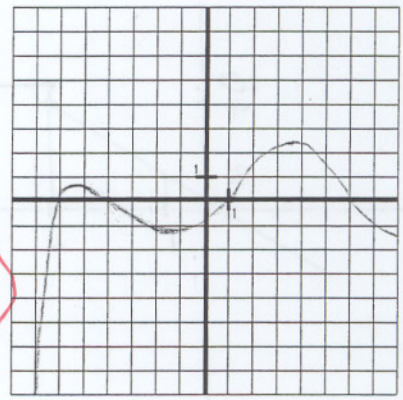
**Math 1B**
**Quiz 2**
**Fri Apr 18, 2008**

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___

# THIS IS A NO CALCULATOR QUIZ

**[6 POINTS]** The graph of $y = f(x)$ is shown to the right.

$y = f(x)$
**WINDOW** $[-8, 8] \times [-8, 8]$



(a) Write the **total** area between the $x$-axis and $y = f(x)$ as an integral or a sum/difference of integrals.
**DO NOT USE ABSOLUTE VALUES (ERIK'S TRICK) OR ROOTS OF POWERS (ANUJ'S TRICK).**

$$-\int_{-8}^{-6} f(x)\,dx + \int_{-6}^{-4} f(x)\,dx - \int_{-4}^{1} f(x)\,dx + \int_{1}^{6} f(x)\,dx - \int_{6}^{8} f(x)\,dx$$

(b) Determine whether $\int_1^8 f(x)dx$ is positive or negative. **Explain briefly.**

POSITIVE

AREA ABOVE X-AXIS ON [1,6]
GREATER THAN
AREA UNDER X-AXIS ON [6,8]

4 POINT EACH TIME
WRITE dx

**[2 POINTS]** Write each expression below **as a single integral**.

(a) $\int_1^8 f(x)dx - \int_3^8 f(x)dx$

$$\int_1^3 f(x)\,dx$$

(b) $\int_4^5 f(x)dx + \int_1^4 f(x)dx$

$$\int_1^5 f(x)\,dx$$

SUBTRACT
YOU DON'T

**[4 POINTS]** Write the definition of "definite integral":

THE DEFINITE INTEGRAL OF $f$ ON $[a,b]$ IS

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

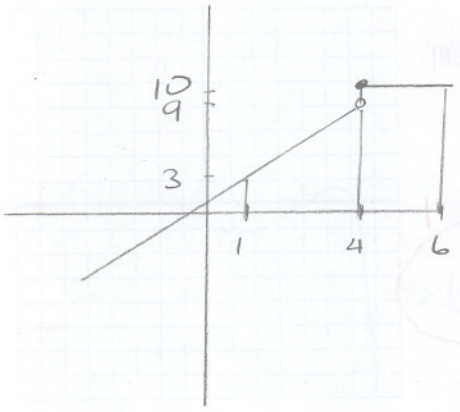
WHERE $\Delta x = \frac{b-a}{n}$ AND $a+(i-1)\Delta x \le x_i^* \le a+i\Delta x$

THE LIMIT MUST EXIST AND BE THE SAME REGARDLESS OF HOW THE $x_i^*$ ARE CHOSEN.

**QUESTIONS ON THE OTHER SIDE]**

# THIS IS A NO CALCULATOR QUIZ

**[4 POINTS]** Let $f(x) = \begin{cases} 2x+1 & \text{if } x < 4 \\ 10 & \text{if } x \geq 4 \end{cases}$. Compute $\int_1^6 f(x)dx$ **using geometry. DO NOT USE ANTI-DERIVATIVES.**




$= \frac{1}{2}(3+9)3 + 10(2)$ (2) (1)

$= 18 + 20$

$= 38$ (1)

**[2 POINTS]** In each sentence below, circle the underlined word which completes the sentence correctly.

If $f(x)$ is decreasing and concave up on $[a, b]$,

(a) a Riemann sum with midpoint evaluation points will be **MORE / LESS** than the area under $f(x)$ on $[a, b]$

(b) a Riemann sum with left-endpoint evaluation points will be **MORE / LESS** than the area under $f(x)$ on $[a, b]$

**[2 POINTS]** **MULTIPLE CHOICE (NO PARTIAL CREDIT)**

Evaluate the Riemann sum of $f(x) = x^2 - 5x$ on $[1, 9]$ using 4 subintervals, if the evaluation points are the midpoints of each subinterval.

| | | | | | | |
|---|---|---|---|---|---|---|
| [A] | 42 | [B] | 40 | [C] | 44 | **LETTER OF CORRECT ANSWER:** B |
| [D] | 38 | [E] | 46 | [F] | 36 | |

**[2 BONUS POINTS]** Sketch the graph of a continuous function $f(x)$ such that the Riemann sum of $f(x)$ on $[a, b]$ using 2 subintervals is more than the area under $f(x)$ on $[a, b]$, regardless of whether the evaluation points are left-endpoints, midpoints or right-endpoints. **HINT: TRY FIRST WITH 1 SUBINTERVAL.**

**[QUESTIONS ON THE OTHER SIDE]**