Math 1B
Quiz 3
Fri Apr 25, 2008

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___

THIS IS A NO CALCULATOR QUIZ

[3 POINTS] Find a value of c that satisfies the conclusion of the Integral Mean Value Theorem if $\int_0^2 3x^2\,dx = 8$.

1 $\frac{1}{2-0}\int_0^2 3x^2\,dx = \frac{1}{2}\cdot 8 = 4$

1 $3x^2 = 4$

$x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$ $\frac{1}{2}$

$\frac{1}{2}$ ONLY $C = \frac{2}{\sqrt{3}}$ IS IN THE INTERVAL $[0,2]$

[9 POINTS] Evaluate each indefinite integral.

(a) $\int \frac{e^x}{1+e^{2x}}\,dx$ $= \int \frac{1}{1+u^2}\,du$ 2

$= \arctan u + C$

$= \arctan e^x + C$ 1

$u = e^x$

$\frac{du}{dx} = e^x$

$\frac{du}{e^x} = dx$

$\frac{e^x}{1+e^{2x}}\frac{du}{e^x} = \frac{e^x}{1+e^{2x}}\,dx$

$\frac{1}{1+e^{2x}}\,du = \frac{1}{1+u^2}\,du$

(b) $\int \frac{2x+3}{x+7}\,dx$

$u = x+7$
$x = u-7$
$du = dx$

$\frac{2x+3}{x+7}\,du = \frac{2x+3}{x+7}\,dx$

$\frac{2(u-7)+3}{u}\,du$

2

$= \frac{2u-11}{u}\,du$

$= \left(2 - \frac{11}{u}\right)du$

$= \int\left(2 - \frac{11}{u}\right)du$

$= 2u - 11\ln|u| + C$

1

$= 2(x+7) - 11\ln|x+7| + C$

$\frac{1}{2}$ $= 2x - 11\ln|x+7| + C$

(c) $\int 3x^3 \sin x^4\,dx$ $= \frac{3}{4}\int \sin u\,du$ 2

$= \frac{3}{4}(-\cos u) + C$

$= -\frac{3}{4}\cos x^4 + C$ 1

$u = x^4$

$\frac{du}{dx} = 4x^3$

$\frac{du}{4x^3} = dx$

$3\cancel{x^3}\sin x^4\,\frac{du}{4\cancel{x^3}} = 3x^3\sin x^4\,dx$

$\frac{3}{4}\sin x^4\,du = \frac{3}{4}\sin u\,du$

[QUESTIONS ON OTHER SIDE]

THIS IS A NO CALCULATOR QUIZ

[3 POINTS] Compute $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{4}{\sqrt{1-x^2}}\, dx$ exactly.

1

$= 4 \arcsin x \Big|_{-\frac{1}{2}}^{\frac{1}{2}}$

1

$= 4\left(\arcsin \frac{1}{2} - \arcsin\left(-\frac{1}{2}\right)\right)$

$= 4\left(\frac{\pi}{6} - -\frac{\pi}{6}\right) = 4\left(\frac{\pi}{3}\right) = \frac{4\pi}{3}$ 1

[3 POINTS] Find all local extrema of $f(x) = \int_{7}^{x} (t^2 - 3t + 2)\, dt$. **<u>SPECIFY IF THE EXTREMA ARE MAXIMA OR MINIMA.</u>**

$\frac{1}{2}$ NEED $f'(x) = 0$

1 $x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$\frac{1}{2}$ $x = 1, 2$

2ND DERIVATIVE TEST

$f''(x) = 2x - 3$

$\frac{1}{2}$ $f''(1) = -1 < 0 \Rightarrow$ MAX

$\frac{1}{2}$ $f''(2) = 1 > 0 \Rightarrow$ MIN

$x = 1$ IS LOCAL MAX

$x = 2$ IS LOCAL MIN

[2 POINTS] **MULTIPLE CHOICE (NO PARTIAL CREDIT)**

$\int_0^1 \left(\sqrt[4]{x} + x\sqrt{x}\right) dx =$

[A]	$\frac{4}{3}$	[B]	$\frac{6}{5}$	[C]	2
[D]	$\frac{5}{3}$	[E]	1	[F]	$\frac{17}{15}$

LETTER OF CORRECT ANSWER: B

[2 BONUS POINTS] Using an appropriate u-substitution, show that $\int_a^1 \frac{1}{x^2+1}\, dx = \int_1^{a^{-1}} \frac{1}{x^2+1}\, dx$ for $a > 0$.