Math 1B
Quiz 5
Thu May 15, 2008

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___

THIS IS A NO CALCULATOR QUIZ

You may or may not need to use the following reduction formulae.

$$\int \sin^n u\, du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\, du$$

$$\int \cos^n u\, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\, du$$

$$\int \sec^n u\, du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\, du, \quad \textit{if } n \neq 1$$

[11 POINTS] Evaluate the following integrals. **<u>YOU MUST NOT USE COMPLEX VALUED FUNCTIONS.</u>**

$$\int \frac{x}{\sqrt{x^2+9}}\, dx$$

$u = x^2+9$
$du = 2x\, dx$
$\frac{1}{2} du = x\, dx$

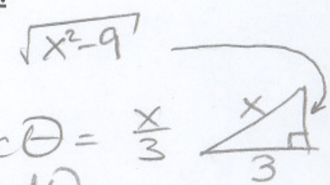
$\int \frac{1}{2}\frac{1}{\sqrt{u}}\, du$ 1

$= \frac{1}{2}\int u^{-\frac{1}{2}}\, du$

$= u^{\frac{1}{2}} + C$ $\frac{1}{2}$

$= \sqrt{x^2+9} + C$ $\frac{1}{2}$ $\frac{1}{2}$

$$\int \frac{1}{\sqrt{x^2-9}}\, dx$$



$x = 3\sec\theta \longrightarrow \sec\theta = \frac{x}{3}$ 1
$dx = 3\sec\theta\tan\theta\, d\theta$

$\int \frac{1}{\cancel{3}\tan\theta}\, \cancel{3}\sec\theta \cancel{\tan\theta}\, d\theta$ $\frac{1}{2}$

$= \int \sec\theta\, d\theta$ $\frac{1}{2}$

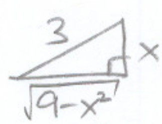
$= \ln|\sec\theta + \tan\theta| + C$ $\frac{1}{2}$

$= \ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right| + C$ 1

$= \ln|x + \sqrt{x^2-9}| + C$ $\frac{1}{2}$ $\frac{1}{2}$

$$\int \frac{1}{x^2\sqrt{9-x^2}}\, dx$$

$u = 3\sin\theta \longrightarrow \sin\theta = \frac{x}{3}$ 1



$du = 3\cos\theta\, d\theta$ $\frac{1}{2}$

$\int \frac{1}{9\sin^2\theta\, \cancel{3\cos\theta}}\, \cancel{3\cos\theta}\, d\theta$

$= \frac{1}{9}\int \csc^2\theta\, d\theta$ $\frac{1}{2}$

$= -\frac{1}{9}\cot\theta + C$ $\frac{1}{2}$

$= -\frac{1}{9}\frac{\sqrt{9-x^2}}{x} + C = -\frac{\sqrt{9-x^2}}{9x} + C$ 1 $\frac{1}{2}$

CONTINUED ON OTHER SIDE

[4 POINTS] Evaluate the following integral.

$$\int \frac{x+15}{x^2-5x-6}\,dx = \int \left(\frac{A}{x-6} + \frac{B}{x+1}\right) dx \quad 1$$

$x+15 = A(x+1) + B(x-6)$ $\frac{1}{2}$

$x=-1:\ 14 = -7B \to B=-2$ $\frac{1}{2}$

$x=6:\ 21 = 7A \to A=3$ $\frac{1}{2}$

$$= \int \left(\frac{3}{x-6} - \frac{2}{x+1}\right) dx$$

$$= 3\ln|x-6| - 2\ln|x+1| + C$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

[3 POINTS] Evaluate the following integral.

$$\int \tan^3 x \sec^3 x\,dx$$

$u = \sec x$ 1

$du = \sec x \tan x\,dx$

$\int \tan^2 x \sec^2 x (\sec x \tan x)\,dx$

$= \int (u^2-1)u^2\,du$ $\frac{1}{2}$

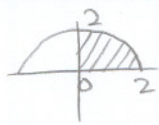
$= \int (u^4 - u^2)\,du$

$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$ $\frac{1}{2}$

$= \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$ $\frac{1}{2}$ $\frac{1}{2}$

[2 POINTS] **MULTIPLE CHOICE (NO PARTIAL CREDIT)**

$\int_0^2 \sqrt{4-x^2}\,dx$ is closest to



HALF OF SEMI-CIRCLE OF RADIUS 2 $= \pi$

[A] 3 [B] 4 [C] 5
[D] 6 [E] 7 [F] 8

LETTER OF CORRECT ANSWER: A

[2 BONUS POINTS] For the integral $\int \tan x \sec^2 x\,dx$, Jim used $u = \tan x$ and got the answer $\frac{1}{2}\tan^2 x + C$, while Kin used $u = \sec x$ and got the answer $\frac{1}{2}\sec^2 x + C$. Show why their answers are equivalent.

$\frac{1}{2}\sec^2 x + C$

$= \frac{1}{2}(\tan^2 x + 1) + C$

$= \frac{1}{2}\tan^2 x + \underbrace{C + \frac{1}{2}}$

$= \frac{1}{2}\tan^2 x + K$