Math 1B
Quiz 5
Thu May 15, 2008

What day of the month is your birthday ? __ __
What are the last 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __

THIS IS A NO CALCULATOR QUIZ

You may or may not need to use the following reduction formulae.

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1}u\cos u + \frac{n-1}{n}\int \sin^{n-2}u\,du$$

$$\int \cos^n u\,du = \frac{1}{n}\cos^{n-1}u\sin u + \frac{n-1}{n}\int \cos^{n-2}u\,du$$

$$\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2}u\tan u + \frac{n-2}{n-1}\int \sec^{n-2}u\,du, \quad if\ n \neq 1$$

[11 POINTS] Evaluate the following integrals. **YOU MUST NOT USE COMPLEX VALUED FUNCTIONS.**

$$\int \frac{x}{\sqrt{x^2+4}}\,dx$$

$u = x^2+4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\int \frac{1}{2}\frac{1}{\sqrt{u}}\,du$ 1

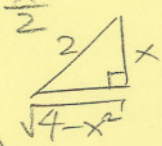
$= \int \frac{1}{2}u^{-\frac{1}{2}}\,du$

$= u^{\frac{1}{2}} + C$ $\frac{1}{2}$

$= \sqrt{x^2+4} + C$ $\frac{1}{2}$ $\frac{1}{2}$

$$\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$$

$x = 2\sin\theta \longrightarrow \sin\theta = \frac{x}{2}$ 1



$dx = 2\cos\theta\,d\theta$ $\frac{1}{2}$

$\int \frac{1}{4\sin^2\theta\,\cancel{2\cos\theta}}\cancel{2\cos\theta}\,d\theta$

$= \frac{1}{4}\int \csc^2\theta\,d\theta$ $\frac{1}{2}$

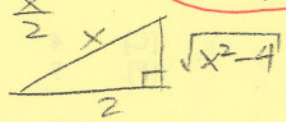
$= -\frac{1}{4}\cot\theta + C$ $\frac{1}{2}$

$= -\frac{1}{4}\frac{\sqrt{4-x^2}}{x} + C$

$= -\frac{\sqrt{4-x^2}}{4x} + C$ $\frac{1}{2}$ 1

$$\int \frac{1}{\sqrt{x^2-4}}\,dx$$

$x = 2\sec\theta \longrightarrow \sec\theta = \frac{x}{2}$ 1



$dx = 2\sec\theta\tan\theta\,d\theta$

$\int \frac{1}{\cancel{2\tan\theta}}\cancel{2}\sec\theta\,\cancel{\tan\theta}\,d\theta$ $\frac{1}{2}$

$= \int \sec\theta\,d\theta$ $\frac{1}{2}$

$= \ln|\sec\theta + \tan\theta| + C$ $\frac{1}{2}$

$= \ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| + C = \ln\left|x + \sqrt{x^2-4}\right| + C$ $\frac{1}{2}$ $\frac{1}{2}$ 1

CONTINUED ON OTHER SIDE

[4 POINTS] Evaluate the following integral.

$$\int \frac{x+15}{x^2-5x-6}\,dx$$

$$= \int \left(\frac{A}{x-6} + \frac{B}{x+1} \right) dx \quad 1$$

$\frac{1}{2}$

$x + 15 = A(x+1) + B(x-6)$

$x = -1 : 14 = -7B$

$B = -2$ $\frac{1}{2}$

$x = 6 : 21 = 7A$

$A = 3$ $\frac{1}{2}$

$$= \int \left(\frac{3}{x-6} - \frac{2}{x+1} \right) dx$$

$$= 3\ln|x-6| - 2\ln|x+1| + C$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

[3 POINTS] Evaluate the following integral.

$$\int \tan^3 x \sec^3 x \, dx$$

1 $u = \sec x$

$du = \sec x \tan x \, dx$

$$\int \tan^2 \sec^2 x \,(\sec x \tan x)\,dx$$

$$= \int (u^2-1)u^2 \, du \quad \tfrac{1}{2}$$

$$= \int (u^4 - u^2)\, du \quad \tfrac{1}{2}$$

$$= \tfrac{1}{5}u^5 - \tfrac{1}{3}u^3 + C \quad \tfrac{1}{2}$$

$$= \tfrac{1}{5}\sec^5 x - \tfrac{1}{3}\sec^3 x + C \quad \tfrac{1}{2}$$

[2 POINTS] **MULTIPLE CHOICE (NO PARTIAL CREDIT)** ERRATUM

$$\int_0^2 \sqrt{9-x^2}\,dx$$ is closest to

2 ← WAS SUPPOSED TO BE 3

[A]	2	[B]	3	[C]	4
[D]	5	[E]	6	[F]	7

LETTER OF CORRECT ANSWER: ____

[2 BONUS POINTS] For the integral $\int \tan x \sec^2 x \, dx$, Jim used $u = \tan x$ and got the answer $\frac{1}{2}\tan^2 x + C$, while Kin used $u = \sec x$ and got the answer $\frac{1}{2}\sec^2 x + C$. Show why their answers are equivalent.

$$\tfrac{1}{2}\tan^2 x + C$$

$$= \tfrac{1}{2}(\sec^2 x - 1) + C$$

$$= \tfrac{1}{2}\sec^2 x \underbrace{- \tfrac{1}{2} + C}$$

$$= \tfrac{1}{2}\sec^2 x + K$$