Math 1B
Quiz 6
Fri May 30, 2008

What day of the month is your birthday ? __ __
What are the last 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __

THIS IS A NO CALCULATOR QUIZ

[10 POINTS] Determine whether the following integrals converge or diverge. **You must show work which supports your conclusion. You do NOT need to find the value of an integral if it converges.**

$$\int_0^\infty \frac{\cos^2 x}{e^x+1}\,dx$$

1 POINT EACH (TOTAL = 5)

$$0 \le \frac{\cos^2 x}{e^x+1} \le \frac{1}{e^x+1} < \frac{1}{e^x}$$

OR EITHER ONE

$$0 \le \frac{\cos^2 x}{e^x+1} < \frac{\cos^2 x}{e^x} \le \frac{1}{e^x}$$

$$\int_0^\infty \frac{1}{e^x}\,dx = \lim_{N\to\infty} -e^{-x}\Big|_0^N$$

$$= \lim_{N\to\infty}\left(-e^{-N} - -1\right)$$

$$= 0 + 1 = 0$$

$\int_0^\infty \frac{1}{e^x}\,dx$ CONV, SO $\int_0^\infty \frac{\cos^2 x}{e^x+1}\,dx$ CONV

$$\int_2^\infty \frac{\sqrt{x}}{x-1}\,dx$$

1 POINT EACH (TOTAL = 5)

$$\frac{\sqrt{x}}{x-1} > \frac{\sqrt{x}}{x} = \frac{1}{x^{\frac{1}{2}}} > 0$$

$\int_2^\infty \frac{1}{x^{\frac{1}{2}}}\,dx$ DIVERGES $\left(p = \frac{1}{2} < 1\right)$

SO $\int_2^\infty \frac{\sqrt{x}}{x-1}\,dx$ DIVERGES

[10 POINTS] Determine whether the following integrals converge or diverge. Find the value of each integral that converges. **You must show work which supports your conclusion.**

$$\int_{-\infty}^0 x^2 e^{4x}\,dx$$

u		dv
x^2	$+$	e^{4x}
$2x$	$-$	$\frac{1}{4}e^{4x}$
2	$+$	$\frac{1}{16}e^{4x}$
0		$\frac{1}{64}e^{4x}$

1

$$= \lim_{N\to-\infty}\int_N^0 x^2 e^{4x}\,dx$$

2

$$= \lim_{N\to-\infty}\left(\left(\frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x}\right)\right)\Big|_N^0$$

$$= \lim_{N\to-\infty}\left(\frac{1}{32} - \left(\frac{1}{4}N^2 - \frac{1}{8}N + \frac{1}{32}\right)e^{4N}\right)$$

$\frac{1}{2}$

$$= \frac{1}{32} - 0 = \frac{1}{32}$$

$\frac{1}{2}$

1

1

$$\lim_{N\to-\infty}\left(\frac{1}{4}N^2 - \frac{1}{8}N + \frac{1}{32}\right)e^{4N} = \lim_{N\to-\infty}\frac{\frac{1}{4}N^2 - \frac{1}{8}N + \frac{1}{32}}{e^{-4N}} = \lim_{N\to-\infty}\frac{\frac{1}{2}N - \frac{1}{8}}{-4e^{-4N}} = \lim_{N\to-\infty}\frac{\frac{1}{2}}{16e^{-4N}} = 0$$

$\infty \cdot 0$ $\quad \frac{\infty}{\infty}$ $\quad \frac{\infty}{\infty}$

$$\int_1^\infty \frac{x+1}{x^3}\,dx$$

1 POINT EACH (TOTAL = 4)

$$= \lim_{N\to\infty}\int_1^N\left(\frac{1}{x^2} + \frac{1}{x^3}\right)dx$$

$$= \lim_{N\to\infty}\left(-x^{-1} - \frac{1}{2}x^{-2}\right)\Big|_1^N$$

$$= \lim_{N\to\infty}\left(-N^{-1} - \frac{1}{2}N^{-2} + 1 + \frac{1}{2}\right)$$

$$= 0 - 0 + \frac{3}{2} = \frac{3}{2}$$