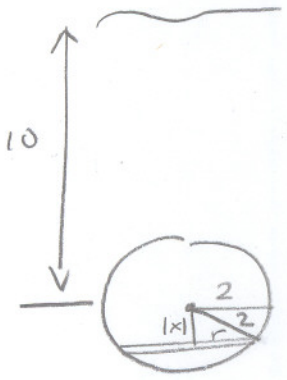
Math 1B
Quiz 8
Fri Jun 13, 2008

What day of the month is your birthday ? ___ ___
What are the last 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___

[9 POINTS] A spherical tank of radius 2 feet containing water is buried underground, so that its center is 10 feet below ground level. Find the work done in pumping the water to ground level if the tank is half full.
You must give an exact answer, not a decimal approximation generated by fnInt.



$x=-2 \quad d=8$
$x=0 \quad d=10$
$x=2 \quad d=12$
$d = x+10$
$r^2 = 4-x^2$

$$\int_0^2 8\pi(4-x^2)(x+10)\,dx$$

$$= 8\pi \int_0^2 (40+4x-10x^2-x^3)\,dx$$

$$= 8\pi \left(40x+2x^2-\frac{10}{3}x^3-\frac{1}{4}x^4\right)\Big|_0^2$$

$$= 8\pi \left(80+8-\frac{80}{3}-4\right)$$

$$= \frac{172}{3}8\pi$$

$$= \frac{172}{3}62.4\pi$$

$$= 3577.6\pi \text{ LB-FT}$$

OR

$$\int_0^2 8\pi(4x-x^2)(12-x)\,dx$$

$$= 8\pi \int_0^2 (48x-16x^2+x^3)\,dx$$

$$= 8\pi \left(24x^2-\frac{16}{3}x^3+\frac{1}{4}x^4\right)\Big|_0^2$$

$$= 8\pi \left(96-\frac{128}{3}+4\right)$$

$$= \frac{172}{3}8\pi = \frac{172}{3}62.4\pi$$

$$= 3577.6\pi \text{ LB-FT}$$

OR

$$\int_{-2}^0 8\pi(4-x^2)(10-x)\,dx$$

$$= 8\pi \int_{-2}^0 (40-4x-10x^2+x^3)\,dx$$

$$= 8\pi \left(40x-2x^2-\frac{10}{3}x^3+\frac{1}{4}x^4\right)\Big|_{-2}^0$$

$$= 8\pi \left(-\left(-80-8+\frac{80}{3}+4\right)\right)$$

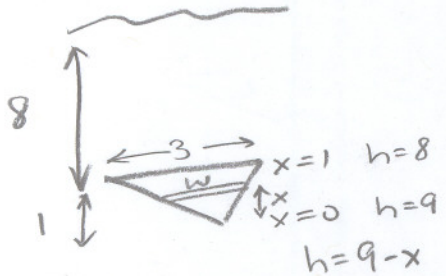
$$= \frac{172}{3}8\pi$$

$$= \frac{172}{3}62.4\pi$$

$$= 3577.6\pi \text{ LB-FT}$$

[7 POINTS] Find the hydrostatic force on the window of an aquarium if the window is a triangle of height 1 foot and base 3 feet with the base up and 8 feet below the surface of the water.

You must give an exact answer, not a decimal approximation generated by fnInt.



$\frac{w}{x} = \frac{3}{1}$

$w = 3x$

$$\int_0^1 \delta (3x)(9-x)\,dx$$

$$= 3\delta \int_0^1 (9x - x^2)\,dx$$

$$= 3\delta \left(\frac{9}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1$$

$$= 3\delta \left(\frac{9}{2} - \frac{1}{3}\right)$$

$$= 3\delta \left(\frac{25}{6}\right)$$

$$= \frac{25}{2}\delta$$

$$= 780 \text{ LB}$$

OR

$$\int_0^1 \delta (3-3x)(8+x)\,dx$$

$$= \delta \int_0^1 (24 - 21x - 3x^2)\,dx$$

$$= \delta \left(24x - \frac{21}{2}x^2 - x^3\right)\Big|_0^1$$

$$= \delta \left(24 - \frac{21}{2} - 1\right) = \frac{25}{2}\delta = 780 \text{ LB}$$

[4 POINTS] Find the length of the curve $y = \frac{1}{8}x^4 + \frac{1}{4x^2}$ on $[1, \pi]$.

You must give an exact answer, not a decimal approximation generated by fnInt.

$$\int_1^\pi \sqrt{1 + \left(\frac{1}{2}x^3 - \frac{1}{2}x^{-3}\right)^2}\,dx$$

$$= \int_1^\pi \sqrt{1 + \frac{1}{4}x^6 - \frac{1}{2} + \frac{1}{4}x^{-6}}\,dx$$

$$= \int_1^\pi \sqrt{\frac{1}{4}x^6 + \frac{1}{2} + \frac{1}{4}x^{-6}}\,dx$$

$$= \int_1^\pi \left(\frac{1}{2}x^3 + \frac{1}{2}x^{-3}\right)dx$$

$$= \left(\frac{1}{8}x^4 - \frac{1}{4}x^{-2}\right)\Big|_1^\pi$$

$$= \frac{\pi^4}{8} - \frac{1}{4\pi^2} - \left(\frac{1}{8} - \frac{1}{4}\right) = \frac{\pi^4}{8} - \frac{1}{4\pi^2} + \frac{1}{8} = \frac{\pi^6 + \pi^2 - 2}{8\pi^2}$$