

[10 POINTS] Find the area under the curve $f(x) = 3x^2 + 2x$ from $x = 1$ to $x = 4$ using the limit of the right handed sums.

$$\Delta x = \frac{4-1}{n} = \frac{3}{n}$$

$$x_i = 1 + \frac{3i}{n}$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(1 + \frac{3i}{n}\right) \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[3\left(1 + \frac{3i}{n}\right)^2 + 2\left(1 + \frac{3i}{n}\right) \right] \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left(3 + \frac{18i}{n} + \frac{27i^2}{n^2} + 2 + \frac{6i}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left(5 + \frac{24i}{n} + \frac{27i^2}{n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(5n + \frac{24}{n} \frac{n(n+1)}{2} + \frac{27}{n^2} \frac{n(n+1)(2n+1)}{6} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(5n + 12(n+1) + \frac{9}{2} (n+1)(2n+1) \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{2n^2} (10n^2 + 24n(n+1) + 9(n+1)(2n+1))$$

$$= \lim_{n \rightarrow \infty} \frac{3}{2n^2} (52n^2 + 51n + 9)$$

$$= \frac{3 \cdot 52}{2} = 78$$

$$= 78$$