Math 1B
Group Quiz 2
Wed Jan 23, 2008

Names: BLUE
GREEN

THIS IS A NO CALCULATOR QUIZ

[5 POINTS] Identify the following as the limit of a Riemann sum and evaluate.

$$\lim_{n\to\infty}\frac{2}{n}\left[\frac{1}{1+2/n}+\frac{1}{1+4/n}+\cdots+\frac{1}{3}\right] = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{1+\frac{2i}{n}}\cdot\frac{2}{n} \quad \tfrac{1}{2}$$

$$f(a+i\Delta x) = \frac{1}{1+\frac{2i}{n}} \qquad \Delta x = \frac{2}{n} \quad \tfrac{1}{2}$$

$$f\left(a+\frac{2i}{n}\right) = \frac{1}{1+\frac{2i}{n}} \qquad a = 1 \quad \tfrac{1}{2}$$

$$f\left(1+\frac{2i}{n}\right) = \frac{1}{1+\frac{2i}{n}} \qquad f(x) = \frac{1}{x} \quad \tfrac{1}{2}$$

$$\Delta x = \frac{b-a}{n}$$

$$\frac{2}{n} = \frac{b-1}{n}$$

$$b = 3 \quad \tfrac{1}{2}$$

$$= \int_1^3 \frac{1}{x}\,dx \quad 1$$

$$= \ln|x| \Big|_1^3 \quad 1$$

$$= \ln|3| - \ln|1|$$

$$= \ln 3 \quad \tfrac{1}{2}$$

$-\frac{1}{2}$ IF MISSING ABSOLUTE VALUE IN $\ln|x|$

[4 POINTS] Find the average value of the function $f(x) = \dfrac{x^2-3}{x}$ on the interval [1, 4].

$$\frac{1}{4-1}\int_1^4 \frac{x^2-3}{x}\,dx$$

$$= \frac{1}{3}\int_1^4 \left(x - \frac{3}{x}\right) dx \quad \tfrac{1}{2}\ \tfrac{1}{2}$$

$$= \frac{1}{3}\left(\frac{1}{2}x^2 - 3\ln|x|\right)\Big|_1^4 \quad 1\ 1$$

$$= \frac{1}{6}x^2 - \ln|x| \Big|_1^4$$

$$= \left(\frac{16}{6} - \ln|4|\right) - \left(\frac{1}{6} - \ln|1|\right)$$

$$= \frac{15}{6} - \ln 4 = \frac{5}{2} - \ln 4 \quad 1$$

$-\frac{1}{2}$ IF NO ABSOLUTE VALUES INSIDE $\ln|x|$

$-\frac{1}{2}$ IF ANY PART OF INTEGRAL NOTATION WRONG (eg. MISSING LIMITS, MISSING dx)

[1 POINT] Circle the **only** integral below to which the Fundamental Theorem of Calculus applies.

MUST BE CONTINUOUS

$$\int_0^5 \ln x\,dx \qquad \int_0^{\frac{\pi}{2}} \tan x\,dx \qquad \int_0^1 \frac{1}{x^2-4}\,dx \qquad \int_{-1}^1 \frac{1}{x^2}\,dx$$

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