

[2 POINTS] Make the indicated substitution for an unspecified function $f(x)$.

$$u = x^3 \text{ for } \int_2^3 x^2 f(x^3) dx$$

$$\begin{aligned} u &= x^3 \\ \frac{du}{dx} &= 3x^2 \\ du &= 3x^2 dx \\ \frac{1}{3} du &= x^2 dx \end{aligned}$$

$$\begin{aligned} x=2 &\rightarrow u=8 \\ x=3 &\rightarrow u=27 \end{aligned}$$

$\frac{1}{2}$ POINT EACH

$$\frac{1}{3} \int_8^{27} f(u) du$$

[6 POINTS] Evaluate the following integrals algebraically. NO CREDIT FOR CALCULATOR-ONLY ANSWERS.

(a) $\int \frac{3-2x}{x+5} dx$

$$\begin{aligned} u &= x+5 \rightarrow x = u-5 \\ du &= dx \\ \int \frac{3-2(u-5)}{u} du \\ &= \int \frac{13-2u}{u} du \\ &= \int \left(\frac{13}{u} - 2 \right) du \\ &= 13 \ln|u| - 2u + C \\ &= 13 \ln|x+5| - 2(x+5) + C \\ &= 13 \ln|x+5| - 2x + C \end{aligned}$$

$-\frac{1}{2}$ IF MISSING "+C"

(b) $\int_0^{\ln \sqrt{3}} \frac{e^x}{e^{2x}+1} dx$

$$\begin{aligned} u &= e^x \quad x=0 \rightarrow u=1 \\ \frac{du}{dx} &= e^x \quad x=\ln \sqrt{3} \rightarrow u=\sqrt{3} \\ \frac{du}{e^x} &= dx \\ \frac{e^x}{e^{2x}+1} \frac{du}{e^x} &= \frac{e^x}{e^{2x}+1} dx \\ \int_1^{\sqrt{3}} \frac{1}{u^2+1} du &= \tan^{-1} u \Big|_1^{\sqrt{3}} \\ &= \tan^{-1} \sqrt{3} - \tan^{-1} 1 \\ &= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12} \end{aligned}$$

$\frac{1}{2}$ POINT EACH

[2 POINTS] Use Simpson's Rule to estimate $\int_1^3 f(x) dx$ from the given data.

x	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6
f(x)	2.3	2.4	2.7	3.1	3.5	4.1	4.8	5.3	5.0	4.5	3.9	3.1	2.4	1.7	0.8	0.1	0.0	0.1	0.8

$$\begin{aligned} S_{10} &= \frac{1}{3} (0.2) (4.1 + 4(4.8) + 2(5.3) + 4(5.0) + \dots \\ &\quad + 4(0.8) + 0.1) \\ &= 6.73 \end{aligned}$$

DECIMAL ANSWER NOT ACCEPTABLE