Math 1B
Group Quiz 6
Wed Feb 20, 2008

Names: ____________________

____

____

____

[4 POINTS] Compute the arc length of $y = \dfrac{x^5}{4} + \dfrac{1}{15x^3}$ over the interval $[1, 2]$ exactly.

NO DECIMAL APPROXIMATIONS WILL BE ACCEPTED.

$$\int_1^2 \sqrt{1 + \left(\tfrac{5}{4}x^4 - \tfrac{1}{5}x^{-4}\right)^2}\, dx \quad 1$$

$$= \int_1^2 \sqrt{1 + \tfrac{25}{16}x^8 - \tfrac{1}{2} + \tfrac{1}{25}x^{-8}}\, dx \quad \tfrac{1}{2}$$

$$= \int_1^2 \sqrt{\tfrac{25}{16}x^8 + \tfrac{1}{2} + \tfrac{1}{25}x^{-8}}\, dx \quad \tfrac{1}{2}$$

$$= \int_1^2 \left(\tfrac{5}{4}x^4 + \tfrac{1}{5}x^{-4}\right) dx \quad \tfrac{1}{2}$$

$$= \tfrac{1}{4}x^5 - \tfrac{1}{15}x^{-3}\Big|_1^2 \quad \tfrac{1}{2}$$

$$= \left(8 - \tfrac{1}{120}\right) - \left(\tfrac{1}{4} - \tfrac{1}{15}\right) \quad \tfrac{1}{2}$$

$$= \frac{937}{120} \quad \tfrac{1}{2}$$

[3 POINTS] Compute the arc length of $y = \int_0^x \sqrt{2t + t^2}\, dt$ over the interval $[0, 2]$ exactly.

NO DECIMAL APPROXIMATIONS WILL BE ACCEPTED.

$$\int_0^2 \sqrt{1 + 2x + x^2}\, dx \quad \tfrac{1}{2}$$

$$= \int_0^2 (1 + x)\, dx \quad \tfrac{1}{2}$$

$$= x + \tfrac{1}{2}x^2 \Big|_0^2 \quad \tfrac{1}{2}$$

$$= 2 + 2$$

$$= 4 \quad \tfrac{1}{2}$$

[3 POINTS] Set up an integral for the surface area if the curve $y = e^{\frac{x}{2}} + e^{-\frac{x}{2}}$ over the interval $[-1, 1]$ is revolved about the x-axis. Find a decimal approximation of the surface area, or evaluate it exactly (for 1 bonus point).

$$\int_{-1}^1 2\pi \left(e^{\frac{x}{2}} + e^{-\frac{x}{2}}\right) \sqrt{1 + \left(\tfrac{1}{2}e^{\frac{x}{2}} - \tfrac{1}{2}e^{-\frac{x}{2}}\right)^2}\, dx \quad 1 \quad \approx 27.3344 \quad 1$$

BONUS:

$$= 2\pi \int_{-1}^1 \left(e^{\frac{x}{2}} + e^{-\frac{x}{2}}\right) \sqrt{1 + \left(\tfrac{1}{4}e^x - \tfrac{1}{2} + \tfrac{1}{4}e^{-x}\right)}\, dx$$

$$= 2\pi \int_{-1}^1 \left(e^{\frac{x}{2}} + e^{-\frac{x}{2}}\right) \sqrt{\tfrac{1}{4}e^x + \tfrac{1}{2} + \tfrac{1}{4}e^{-x}}\, dx$$

$$= 2\pi \int_{-1}^1 \left(e^{\frac{x}{2}} + e^{-\frac{x}{2}}\right)\left(\tfrac{1}{2}e^{\frac{x}{2}} + \tfrac{1}{2}e^{-\frac{x}{2}}\right) dx$$

$$= \pi \int_{-1}^1 \left(e^x + 2 + e^{-x}\right) dx \quad \tfrac{1}{2}$$

$$= \pi \left(e^x + 2x - e^{-x}\right)\Big|_{-1}^1$$

$$= \pi\left(e + 2 - \tfrac{1}{e} - \left(\tfrac{1}{e} - 2 - e\right)\right)$$

$$= 2\pi\left(e + 2 - \tfrac{1}{e}\right) \quad \tfrac{1}{2}$$