Math 1B
Group Quiz 8
Fri Mar 14, 2008

Code: ____________

If the integral converges, find its value. If the integral diverges, show why.

[3 POINTS] $\displaystyle\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$

$u = 4-x$
$du = -dx$

$= \lim_{N\to 4^-} \int_0^N \frac{1}{\sqrt{4-x}}\,dx$ 1

$= \lim_{N\to 4^-} -2\sqrt{4-x}\Big|_0^N$ 1

$= \lim_{N\to 4^-} \left(-2\sqrt{4-N}+4\right)$

$= 4$ 1

[3 POINTS] $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}e^{\sqrt{x}}}\,dx$

$u = \sqrt{x}$
$du = \frac{1}{2\sqrt{x}}dx$

$\frac{1}{2}$

$= \int_0^1 \frac{1}{\sqrt{x}e^{\sqrt{x}}}\,dx + \int_1^\infty \frac{1}{\sqrt{x}e^{\sqrt{x}}}\,dx$

1

$= \lim_{N\to 0^+} \int_N^1 \frac{1}{\sqrt{x}e^{\sqrt{x}}}\,dx + \lim_{M\to\infty} \int_1^M \frac{1}{\sqrt{x}e^{\sqrt{x}}}\,dx$

$= \lim_{N\to 0^+} {}^{\frac{1}{2}} -2e^{-\sqrt{x}}\Big|_N^1 + \lim_{M\to\infty} -2e^{-\sqrt{x}}\Big|_1^M$

$= \lim_{N\to 0^+} \left(-2e^{-1}+2e^{-\sqrt{N}}\right)$

$+ \lim_{M\to\infty} \left(-2e^{-\sqrt{M}}+2e^{-1}\right)$

$= -2e^{-1}+2-0+2e^{-1}$ $\frac{1}{2}$

$= 2$ $\frac{1}{2}$

[4 POINTS] $\displaystyle\int_0^\infty xe^{-x}\,dx$

1

$= \lim_{N\to\infty} \int_0^N xe^{-x}\,dx$

1

$= \lim_{N\to\infty} \left(-xe^{-x}-e^{-x}\right)\Big|_0^N$

u		dv
x	$+$	e^{-x}
1	$-$	$-e^{-x}$
0		e^{-x}

$= \lim_{N\to\infty} \left(-Ne^{-N}-e^{-N}-(0-1)\right)$

$= \lim_{N\to\infty} \frac{-N}{e^N} - 0 + 1$ $\frac{1}{2}$

$= \lim_{N\to\infty} \frac{-1}{e^N} + 1$ 1

$= 0+1$

$= 1$ $\frac{1}{2}$