Math 1B
Group Quiz 9
Wed Mar 19, 2008

Code: _____

THIS IS A NO CALCULATOR QUIZ

[2 POINTS]   Solve the initial value problem $y' = \dfrac{y^2}{x}$, $y(-1) = -1$.

$$\frac{dy}{dx} = \frac{y^2}{x}$$

$$\int \frac{1}{y^2} dy = \int \frac{1}{x} dx + C$$

$$-\frac{1}{y} = \ln|x| + C$$

$$y = -\frac{1}{\ln|x| + C}$$

$$y(-1) = -\frac{1}{\ln|-1| + C} = -1$$

½ POINT EACH

$$-\frac{1}{C} = -1$$

$$C = 1$$

$$y = -\frac{1}{\ln|x| + 1}$$

[3 POINTS]   Use a comparison to determine whether $\displaystyle\int_1^\infty \frac{1}{x^{3/2} + e^{-x}} \, dx$ converges or diverges.

$$0 \le \frac{1}{x^{\frac{3}{2}} + e^{-x}} \le \frac{1}{x^{\frac{3}{2}}}$$ SINCE $e^{-x} > 0$ FOR ALL x

$$\int_1^\infty \frac{1}{x^{\frac{3}{2}}} dx \text{ CONVERGES}$$ SINCE $p = \frac{3}{2} > 1$

SO $$\int_1^\infty \frac{1}{x^{\frac{3}{2}} + e^{-x}} dx \text{ CONVERGES}$$ 1 POINT EACH

[5 POINTS]   Joe takes a chicken breast from a 40°F refrigerator, and places it in a 375°F oven. After 30 minutes, the chicken reaches the ideal internal temperature of 160°F. However, since Joe is busy on his Blackberry at the time, he forgets to remove the chicken until 5 minutes after that. What is the temperature of the chicken at that time ?

YOU MUST SHOW CALCULUS BASED WORK TO RECEIVE CREDIT FOR THIS QUESTION.

$$\frac{dT}{dt} = k(375 - T)   \quad T(0) = 40 \quad T(30) = 160$$ 1

$$\int \frac{1}{375 - T} dT = \int k \, dt + C$$ ½

$$-\ln|375 - T| = kt + C$$ ½

$$375 - T = Ce^{-kt}$$

$$T = 375 - Ce^{-kt}$$ 1

$$T(0) = 375 - C = 40$$

$$C = 335$$ ½

$$T(30) = 375 - 335 e^{-30k} = 160$$

$$215 = 335 e^{-30k}$$

$$k = -\frac{1}{30} \ln \frac{215}{335}$$

$$k = \frac{1}{30} \ln \frac{335}{215}$$

$$k = \frac{1}{30} \ln \frac{67}{43}$$ ½

$$T(35) = 375 - 335 e^{-\left(\frac{1}{30} \ln \frac{67}{43}\right) 35}$$ ½

$$= 375 - 335 \left(\frac{43}{67}\right)^{\frac{7}{6}} = 175.32°F$$ ½