Math 1B
Midterm 1
Fri Feb 1, 2008

Code: _____________________________
I forgot my code, so please charge me two points: (Name) ______________

[35 POINTS] Consider the definite integral $\int_1^5 (4-2x)\,dx$.

(a) Evaluate the integral **using the limit of a right hand sum**.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(4-2\left(1+i\left(\frac{4}{n}\right)\right)\right)\frac{4}{n}$$

$\Delta x = \frac{5-1}{n} = \frac{4}{n}$

$a = 1$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(2 - \frac{8i}{n}\right)$$

$$= \lim_{n\to\infty} \frac{4}{n}\left(2n - \frac{\overset{4}{\cancel{8}}}{\cancel{n}}\frac{\cancel{n}(n+1)}{\cancel{2}}\right)$$

$$= \lim_{n\to\infty} \frac{4}{n}\left(2n-4(n+1)\right)$$

$$= \lim_{n\to\infty} \frac{4}{n}(-2n-4)$$

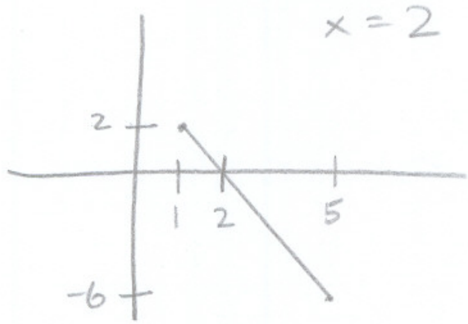
$$= \lim_{n\to\infty} \frac{-8n+16}{n}$$

$$= \frac{-8}{1} = -8$$

(b) Evaluate the integral **geometrically**.

$4-2x=0$

$x=2$



$$\tfrac{1}{2}(2)(1) - \tfrac{1}{2}(6)(3)$$

$$= 1-9$$

$$= -8$$

(c) Evaluate the integral **using the Fundamental Theorem of Calculus (ie. using anti-derivatives)**.

$$4x - x^2 \Big|_1^5$$

$$= (20-25)-(4-1)$$

$$= -5-3$$

$$= -8$$

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[25 POINTS] Let $f(x) = 2x^4 - 5$.

(a) Find the average value of $f(x)$ on $[-1, 4]$.

$$\frac{1}{4--1}\int_{-1}^{4}(2x^4-5)\,dx$$
$$= \frac{1}{5}\left(\frac{2}{5}x^5 - 5x\right)\Big|_{-1}^{4}$$
$$= \frac{1}{5}\left(\frac{2}{5}(4)^5 - 20 - \left(\frac{2}{5}(-1)^5 + 5\right)\right)$$
$$= \frac{1}{5}\left(\frac{2}{5}(4^5+1) - 25\right)$$
$$= \frac{1}{5}\left(\frac{2}{\cancel{5}}\cdot \overset{205}{\cancel{1025}} - 25\right) = \frac{1}{5}\cdot 385 = 77$$

(b) Find a value of c that satisfies the conclusion of the Integral Mean Value Theorem for part (a).

$$2x^4 - 5 = 77$$
$$x^4 = 41$$
$$x = \pm\sqrt[4]{41} \Rightarrow \text{ONLY } x = \sqrt[4]{41} \in [-1, 4]$$

[30 POINTS] Consider the definite integral $\int_1^7 \frac{1}{x}\,dx$.

(a) Using the INTEGRAL program, approximate the integral using the Trapezoidal Rule with n = 50.

$$T_{50} = 1.947083944$$

(b) Using the INTEGRAL program, approximate the integral using Simpson's Rule with n = 10.

$$S_{10} = 1.948519148$$

(c) Find bounds on the error made by the approximation in part (a).

$$\left(\frac{1}{x}\right)'' = \left(-\frac{1}{x^2}\right)' = \frac{2}{x^3} \qquad \left|\frac{2}{x^3}\right| \le 2 \text{ ON } [1, 7]$$
$$|ET_{50}| \le \frac{2(7-1)^3}{12(50)^2} = .0144$$

(d) If the Midpoint Rule is used to approximate the integral, find the number of subintervals needed to guarantee an accuracy of 10^{-7}.

$$\frac{2(7-1)^3}{24n^2} \le 10^{-7}$$
$$\frac{2\cdot 6^3\cdot 10^7}{24} \le n^2$$
$$n \ge \sqrt{\frac{2\cdot 6^3\cdot 10^7}{24}} \cong 13416.4$$

NEED AT LEAST 13,417 SUBINTERVALS

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[45 POINTS] Evaluate the following integrals. **NO CREDIT FOR CALCULATOR-ONLY ANSWERS.**

(a) $\int \frac{x}{\sqrt{1-x^4}}\,dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$\frac{\cancel{x}}{\sqrt{1-x^4}} \frac{du}{2\cancel{x}} = \frac{1}{2}\frac{1}{\sqrt{1-u^2}}\,du$

$\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$

$= \frac{1}{2}\sin^{-1} u + C$

$= \frac{1}{2}\sin^{-1} x^2 + C$

(b) $\int_1^{e^2} \frac{\sqrt{\ln x}}{x}\,dx$

$u = \ln x$

$x = e^2 \Rightarrow u = 2$

$x = 1 \Rightarrow u = 0$

$\frac{du}{dx} = \frac{1}{x}$

$x\,du = dx$

$\frac{\sqrt{\ln x}}{\cancel{x}}\cancel{x}\,du = u^{\frac{1}{2}}du$

$\int_0^2 u^{\frac{1}{2}}du$

$= \frac{2}{3}u^{\frac{3}{2}}\Big|_0^2$

$= \frac{2}{3}(2)^{\frac{3}{2}}$

$= \frac{4}{3}\sqrt{2}$

(c) $\int \frac{(x+1)^2}{\sqrt{x}}\,dx$

$= \int \frac{x^2+2x+1}{x^{\frac{1}{2}}}\,dx$

$= \int \left(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)dx$

$= \frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

(d) $\int_{\frac{2\pi}{3}}^{\pi} \left(3\sin\frac{1}{2}x + \cos 2x - \sec x \tan x\right)dx$

$= -6\cos\frac{1}{2}x + \frac{1}{2}\sin 2x - \sec x \Big|_{\frac{2\pi}{3}}^{\pi}$

$= \left(-6(0) + \frac{1}{2}(0) - (-1)\right)$
$\quad - \left(-6\left(\frac{1}{2}\right) + \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right) - (-2)\right)$

$= 1 - \left(-3 - \frac{\sqrt{3}}{4} + 2\right)$

$= 2 + \frac{\sqrt{3}}{4}$

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[15 POINTS] Let $F(x)=\int_{x^2}^{4} e^{-t^2}\,dt$. Find an equation of the tangent line to $y=F(x)$ at $x=-2$.

$$F(x) = -\int_4^{x^2} e^{-t^2}\,dt$$

$$F'(x) = -e^{-(x^2)^2}\cdot 2x = -2xe^{-x^4}$$

$$F'(-2) = -2(-2)e^{-(-2)^4} = 4e^{-16}$$

$$F(-2) = \int_4^4 e^{-t^2}\,dt = 0$$

$$y - 0 = 4e^{-16}(x - -2)$$

$$y = 4e^{-16}(x+2)$$

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[5 BONUS POINTS] Evaluate $\int_{-3}^{3}\sqrt{36-x^2}\,dx$ **geometrically**. **NO CREDIT FOR DECIMAL APPROXIMATIONS.**

[5 BONUS POINTS] Evaluate $\int_{-5}^{5}\sqrt{36-x^2}\,dx$ **geometrically**. **NO CREDIT FOR DECIMAL APPROXIMATIONS.**