Math 1B
Midterm 3
Wed Mar 12, 2008

Code: ______________________

THIS IS A NO CALCULATOR MIDTERM

Evaluate the following integrals.

[20 POINTS] $\int e^{9x}\sqrt{e^{6x}-4}\,dx$

$u = e^{3x}$

$du = 3e^{3x}\,dx$

$\frac{1}{3}\int u^2\sqrt{u^2-4}\,du$

$= \frac{1}{3}\left(\frac{1}{8}u(2u^2-4)\sqrt{u^2-4} - \frac{1}{8}16\ln\left|u+\sqrt{u^2-4}\right|\right) + C$

$= \frac{1}{12}e^{3x}(e^{6x}-2)\sqrt{e^{6x}-4} - \frac{2}{3}\ln\left|e^{3x}+\sqrt{e^{6x}-4}\right| + C$

[15 POINTS] $\int x^5 \sin x^3\,dx$

$u = x^3$

$du = 3x^2\,dx$

$\frac{1}{3}\int u\sin u\,du$

$= \frac{1}{3}(\sin u - u\cos u) + C$

$= \frac{1}{3}\sin x^3 - \frac{1}{3}x^3\cos x^3 + C$

[BONUS QUESTIONS ON OTHER SIDE]

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[30 POINTS] $\displaystyle\int \frac{x^5 - x}{x^4 + 2x^3 + 2x^2}\,dx = \int \frac{x^4 - 1}{x^3 + 2x^2 + 2x}\,dx = \int \left(x - 2 + \frac{2x^2 + 4x - 1}{x(x^2 + 2x + 2)}\right) dx$

$$\begin{array}{r|l} & x - 2 \\ x^3 + 2x^2 + 2x & x^4 \qquad\qquad -1 \\ & x^4 + 2x^3 + 2x^2 \\ \hline & -2x^3 - 2x^2 \\ & -2x^3 - 4x^2 - 4x \\ \hline & 2x^2 + 4x - 1 \end{array}$$

$$\frac{2x^2 + 4x - 1}{x(x^2 + 2x + 2)} = \frac{A}{x} + \frac{B(2x+2) + C}{x^2 + 2x + 2}$$

$$2x^2 + 4x - 1 = A(x^2 + 2x + 2) + Bx(2x + 2) + Cx$$

$x = 0: \ -1 = 2A \Rightarrow A = -\frac{1}{2}$

$x = -1: \ -3 = A - C \Rightarrow C = A + 3 = \frac{5}{2}$

COEF OF x^2: $\ 2 = A + 2B \Rightarrow B = \frac{1}{2}(2 - A) = \frac{1}{2}\frac{5}{2} = \frac{5}{4}$

$$\int \left(x - 2 - \frac{\frac{1}{2}}{x} + \frac{\frac{5}{4}(2x+2) + \frac{5}{2}}{x^2 + 2x + 2}\right) dx$$

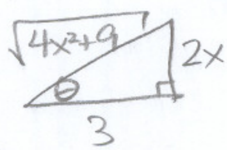
$$= \frac{1}{2}x^2 - 2x - \frac{1}{2}\ln|x| + \frac{5}{4}\ln(x^2 + 2x + 2) + \frac{5}{2}\int \frac{1}{(x+1)^2 + 1}\,dx$$

$$= \frac{1}{2}x^2 - 2x - \frac{1}{2}\ln|x| + \frac{5}{4}\ln(x^2 + 2x + 2) + \frac{5}{2}\tan^{-1}(x+1) + C$$

[15 POINTS] $\int \sqrt{x}(\ln x)^3\,dx$ $= \frac{2}{3}x^{\frac{3}{2}}(\ln x)^3 - \frac{4}{3}x^{\frac{3}{2}}(\ln x)^2 + \frac{16}{9}x^{\frac{3}{2}}\ln x - \frac{32}{27}x^{\frac{3}{2}} + C$

u	dv
$(\ln x)^3$	$+\,x^{\frac{1}{2}}$
$3(\ln x)^2\frac{1}{x}$	$\frac{2}{3}x^{\frac{3}{2}}$
$3(\ln x)^2$	$-\,\frac{2}{3}x^{\frac{1}{2}}$
$6(\ln x)\frac{1}{x}$	$\frac{4}{9}x^{\frac{3}{2}}$
$6\ln x$	$+\,\frac{4}{9}x^{\frac{1}{2}}$
$\frac{6}{x}$	$\frac{8}{27}x^{\frac{3}{2}}$
6	$-\,\frac{8}{27}x^{\frac{1}{2}}$
0	$\frac{16}{81}x^{\frac{3}{2}}$

[25 POINTS] $\int x^2\sqrt{4x^2+9}\,dx$



$= \frac{1}{16}x(4x^2+9)^{\frac{3}{2}} - \frac{9}{32}x(4x^2+9)^{\frac{1}{2}} + \frac{81}{16}\ln\left(\sqrt{4x^2+9} + 2x\right) + C$

$x = \frac{3}{2}\tan\theta$

$dx = \frac{3}{2}\sec^2\theta\,d\theta$

$$\int \frac{9}{4}\tan^2\theta \cdot 3\sec\theta \cdot \frac{3}{2}\sec^2\theta\,d\theta$$

$$= \frac{81}{8}\int \tan^2\theta\sec^3\theta\,d\theta$$

$$= \frac{81}{8}\int (\sec^2\theta - 1)\sec^3\theta\,d\theta$$

$$= \frac{81}{8}\left(\int \sec^5\theta\,d\theta - \int\sec^3\theta\,d\theta\right)$$

$$= \frac{81}{8}\left(\frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{4}\int\sec^3\theta\,d\theta - \int\sec^3\theta\,d\theta\right)$$

$$= \frac{81}{8}\left(\frac{1}{4}\sec^3\theta\tan\theta - \frac{1}{4}\int\sec^3\theta\,d\theta\right)$$

$$= \frac{81}{8}\left(\frac{1}{4}\sec^3\theta\tan\theta - \frac{1}{4}\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|\right)\right) + C$$

$$= \frac{81}{32}\sec^3\theta\tan\theta - \frac{81}{64}\sec\theta\tan\theta + \frac{81}{16}\ln|\sec\theta + \tan\theta| + C$$

[25 POINTS] $\displaystyle\int_{\frac{\pi}{3}}^{\frac{3\pi}{2}} \sin^5 x \cos^5 x\, dx$

$u = \cos x \qquad x = \frac{\pi}{3} \to u = \frac{1}{2}$

$du = -\sin x\, dx \qquad x = \frac{3\pi}{2} \to u = 0$

$$\int_{\frac{\pi}{3}}^{\frac{3\pi}{2}} \sin^4 x \cos^5 x \sin x\, dx$$

$$= -\int_{\frac{1}{2}}^{0} (1-u^2)^2 u^5\, du$$

$$= -\int_{\frac{1}{2}}^{0} (1-2u^2+u^4) u^5\, du$$

$$= -\int_{\frac{1}{2}}^{0} (u^5 - 2u^7 + u^9)\, du$$

$$= -\left(\frac{1}{6}u^6 - \frac{1}{4}u^8 + \frac{1}{10}u^{10}\right)\Big|_{\frac{1}{2}}^{0}$$

$$= -\left(\frac{1}{6}\cdot\frac{1}{64} - \frac{1}{4}\cdot\frac{1}{256} + \frac{1}{10}\cdot\frac{1}{1024}\right)$$

$$= -\frac{1}{1024}\left(\frac{16}{6} - 1 + \frac{1}{10}\right) = -\frac{1}{1024}\cdot\frac{53}{30} = -\frac{53}{30720}$$

[20 POINTS] $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{-3x} \cos 4x\, dx$

u		dv
$\cos 4x$	$+$	e^{-3x}
$-4\sin 4x$	$-$	$-\frac{1}{3}e^{-3x}$
$-16\cos 4x$	$\pm$	$\frac{1}{9}e^{-3x}$

$$\int e^{-3x}\cos 4x\, dx = -\frac{1}{3}e^{-3x}\cos 4x + \frac{4}{9}e^{-3x}\sin 4x - \frac{16}{9}\int e^{-3x}\cos 4x\, dx$$

$$\frac{25}{9}\int e^{-3x}\cos 4x\, dx = -\frac{1}{3}e^{-3x}\cos 4x + \frac{4}{9}e^{-3x}\sin 4x + C$$

$$\int e^{-3x}\cos 4x\, dx = -\frac{3}{25}e^{-3x}\cos 4x + \frac{4}{25}e^{-3x}\sin 4x + C$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{-3x}\cos 4x\, dx = -\frac{3}{25}e^{-\frac{3\pi}{2}} - \frac{3}{25}e^{-\frac{3\pi}{4}}$$