Math 1B
Quiz 2
Fri Jan 18, 2008

Name: ALL COLORS
Code from Greensheet Take Home Quiz:
(month * day * year of birth) _____

THIS IS A NO CALCULATOR QUIZ

[3 POINTS] Complete the definition of "definite integral":

The definite integral OF f ON $[a,b]$ IS $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$

WHERE $\Delta x = \frac{b-a}{n}$ AND $a+(i-1)\Delta x \le x_i^* \le a + i\Delta x$.

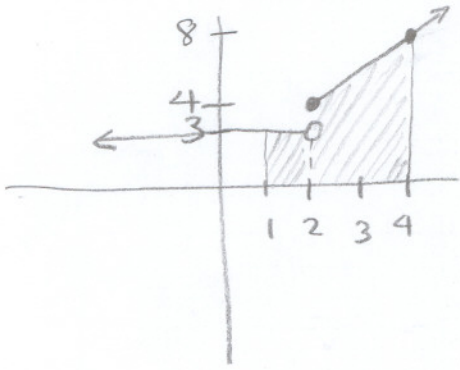
THE LIMIT MUST EXIST AND BE THE SAME REGARDLESS OF THE CHOICE OF THE x_i^*.

$\frac{1}{2}$ POINT EACH

[5 POINTS] Let $f(x) = \begin{cases} 3 & \text{if } x < 2 \\ 2x & \text{if } x \ge 2 \end{cases}$.

(a) Compute $\int_1^4 f(x)dx$ **geometrically**. (It might be helpful, but not necessary, to sketch a graph of $f(x)$.)

PART (a): NO POINTS IF INTEGRAL COMPUTED BY ANTI-DERIVATIVE



$$\int_1^4 f(x)\,dx = 3\times 1 + \tfrac{1}{2}(4+8)2 = 3 + 12 = 15$$

(b) Find the average value of $f(x)$ on the interval $[1, 4]$.

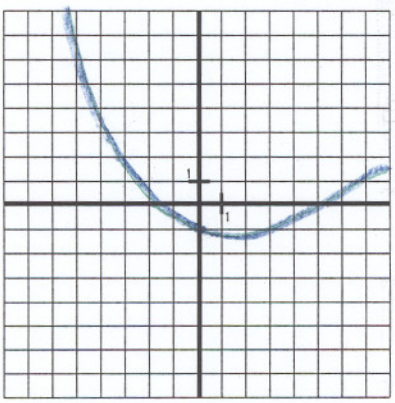
$$\frac{1}{4-1}\int_1^4 f(x)\,dx = \tfrac{1}{3}(15) = 5$$

[QUESTIONS ON THE OTHER SIDE]

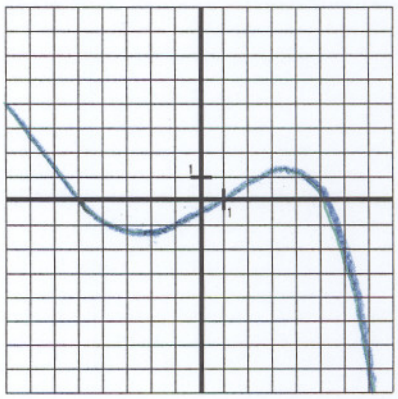
THIS IS A NO CALCULATOR QUIZ

THE GRAPHS OF $y = f(x)$ AND $y = g(x)$ SHOWN BELOW WILL BE USED FOR ALL QUESTIONS ON THIS PAGE.

$y = f(x)$
WINDOW $[-8, 8] \times [-8, 8]$



$y = g(x)$
WINDOW $[-8, 8] \times [-8, 8]$



[5 POINTS] Write each **total** area below as an integral or a sum/difference of integrals.

(a) The area below the x-axis and above $y = f(x)$

$$-\int_{-2}^{5} f(x)\,dx$$

(b) The area between the x-axis and $y = g(x)$ for $-5 \le x \le 5$

$$-\int_{-5}^{1} g(x)\,dx + \int_{1}^{5} g(x)\,dx$$

← $\frac{1}{2}$ POINT EACH →

+ $\frac{1}{2}$ POINT FOR dx (MUST APPEAR IN ALL INTEGRAND)
− $\frac{1}{2}$ POINT IF WRONG FUNCTION IN INTEGRAND
− $\frac{1}{2}$ POINT IF NOT IN INTEGRAL ($\int$) FORM

[4 POINTS] Determine whether $\int_1^8 g(x)dx$ is positive or negative. **Explain briefly.**

PINK/BLUE: NEGATIVE (2) – AREA ABOVE X-AXIS FROM $x=1$ TO $x=5$ ($\frac{1}{2}$) LESS THAN ($\frac{1}{2}$) AREA BELOW X-AXIS FROM $x=5$ TO $x=8$ (1)

WHITE/YELLOW: POSITIVE (2) – AREA ABOVE X-AXIS ON $[1,5]$ ($\frac{1}{2}$) MORE THAN ($\frac{1}{2}$) AREA BELOW ON $[5,8]$ (1)

[3 POINTS] Write each expression below **as a single integral**.

(a) $\int_1^8 f(x)dx + \int_8^3 f(x)dx$

$$\int_1^3 f(x)\,dx$$

(b) $\int_{-3}^{5} f(x)dx + \int_5^7 f(x)dx$

$$\int_{-3}^{7} f(x)\,dx$$

(c) $\int_{-2}^{6} f(x)dx - \int_2^6 f(x)dx$

$$\int_{-2}^{2} f(x)\,dx$$

← $\frac{1}{2}$ POINT EACH →

− $\frac{1}{2}$ POINT IF MISSING dx
− $\frac{1}{2}$ POINT IF NOT IN INTEGRAL FORM
− $\frac{1}{2}$ POINT IF WRONG FUNCTION IN INTEGRAND

[QUESTIONS ON THE OTHER SIDE]