Math 1B
Quiz 3
Fri Jan 25, 2008

Code: ALL COLORS
I forgot my code, so please charge me two points: (Name) ______

THIS IS A NO CALCULATOR QUIZ

[3 POINTS] Complete the definition of "definite integral":

The definite integral OF f ON $[a, b]$ IS $\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$ $\frac{1}{2}$ $\frac{1}{2}$

$\frac{1}{2}$ WHERE $\Delta x = \frac{b-a}{n}$ AND $a + (i-1)\Delta x \le x_i^* \le a + i\Delta x$. $\frac{1}{2}$

$\frac{1}{2}$ THE LIMIT MUST EXIST AND BE THE SAME REGARDLESS

OF THE CHOICE OF THE x_i^*. $\frac{1}{2}$

[6 POINTS] Compute each definite integral exactly.

(a) $\displaystyle\int_1^4 \left(\sqrt[3]{x} - \frac{1}{x\sqrt{x}}\right) dx$

$= \int_1^4 (x^{\frac{1}{3}} - x^{-\frac{3}{2}})\,dx$

$1 \quad 1$

$= \frac{3}{4}x^{\frac{4}{3}} + 2x^{-\frac{1}{2}} \Big|_1^4$

$= \left(\frac{3}{4}4^{\frac{4}{3}} + 2\left(\frac{1}{2}\right)\right) - \left(\frac{3}{4} + 2\right)$ $\frac{1}{2}$

$= 3\sqrt[3]{4} - \frac{7}{4}$ $\frac{1}{2}$

(b) $\displaystyle\int_{\pi}^{\frac{3\pi}{2}} (3\sin x + 2\cos x)\,dx$

$1 \quad 1$

$= -3\cos x + 2\sin x \Big|_{\pi}^{\frac{3\pi}{2}}$

$= (0 - 2) - (3 + 0)$ $\frac{1}{2}$

$= -5$ $\frac{1}{2}$

[QUESTIONS ON OTHER SIDE]

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[2 POINTS] Find the derivative of $f(x)=\int_{e^{3x}}^{4}\csc t\,dt$.

$-3e^{3x}\csc e^{3x}$ $\frac{1}{2}$ POINT EACH

[9 POINTS] Evaluate each indefinite integral.

(a) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$u=\sqrt{x}$ $\frac{1}{2}$

$\frac{du}{dx}=\frac{1}{2\sqrt{x}}$ $\frac{1}{2}$

$2\sqrt{x}\,du=dx$

$\frac{\sin\sqrt{x}}{\cancel{\sqrt{x}}}2\cancel{\sqrt{x}}\,du=\frac{\sin\sqrt{x}}{\sqrt{x}}dx$

$\int 2\sin u\,du$ 1 $=-2\cos u+C$ $\frac{1}{2}$

$=-2\cos\sqrt{x}+C$ $\frac{1}{2}$

(b) $\int \frac{6}{x(1+\ln x)^3}\,dx$

$u=1+\ln x$ $\frac{1}{2}$

$\frac{du}{dx}=\frac{1}{x}$ $\frac{1}{2}$

$x\,du=dx$

$\frac{6}{x(1+\ln x)^3}x\,du=\frac{6}{x(1+\ln x)^3}dx$

$\int\frac{6}{u^3}du$ 1 $=\int 6u^{-3}du$

$=\frac{6}{-2}u^{-2}+C$ $\frac{1}{2}$

$=-3(1+\ln x)^{-2}+C$ $\frac{1}{2}$

(c) $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$u=\sin^{-1}x$ $\frac{1}{2}$

$\frac{du}{dx}=\frac{1}{\sqrt{1-x^2}}$ $\frac{1}{2}$

$\sqrt{1-x^2}\,du=dx$

$\frac{\sin^{-1}x}{\cancel{\sqrt{1-x^2}}}\cancel{\sqrt{1-x^2}}\,du=\frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$

$\int u\,du$ 1 $=\frac{1}{2}u^2+C$ $\frac{1}{2}$

$=\frac{1}{2}(\sin^{-1}x)^2+C$ $\frac{1}{2}$

$-\frac{1}{2}$ POINT EACH TIME "+C" IS LEFT OFF

[QUESTIONS ON OTHER SIDE]