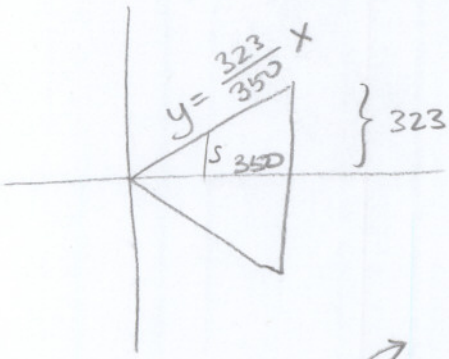
Math 1B
Quiz 4
Fri Feb 8, 2008

Code: _____
I forgot my code, so please charge me two points: (Name) _____

[6 POINTS] The Luxor Hotel in Las Vegas is modeled after the pyramids of Egypt. The building is 350 feet high and its base is a square with sides of 646 feet. Find the volume of the building using an integral. **(DO NOT USE fnInt.)**



$A(x) = (2s)^2 = 4s^2$ 1

$s = \frac{323}{350}x$ 1

$A(x) = 4\left(\frac{323}{350}x\right)^2$

$$\int_0^{350} 4\left(\frac{323}{350}x\right)^2 dx \quad 1$$

$$= 4\left(\frac{323}{350}\right)^2 \int_0^{350} x^2\,dx$$

$$= 4\left(\frac{323}{350}\right)^2 \left(\frac{1}{3}x^3\right)\Big|_0^{350} \quad 1$$

$$= \frac{4}{3}\,\frac{323^2}{350^2}\cdot 350^3$$

$$= 48{,}686{,}866\tfrac{2}{3}\ \text{ft}^3 \quad 1$$

OR

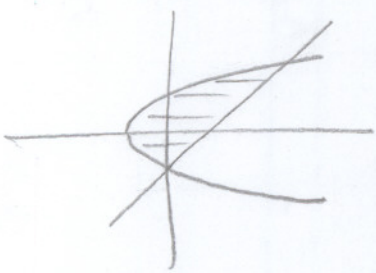
$$\int_0^{350} 4\left(323 - \frac{323}{350}x\right)^2 dx \quad 1$$

$u = 323 - \frac{323}{350}x$

SUBSTITUTION GIVES SAME INTEGRAL

NO CREDIT IF YOU USED $V = \frac{1}{3}Ah$ OR $\frac{1}{3}b^2h$

[5 POINTS] Find the area of the region bounded by the curves $y^2 = x+1$ and $y = \frac{1}{2}x - 1$. **(DO NOT USE fnInt.)**



$x = y^2 - 1 \qquad x = 2(y+1)$

$y^2 - 1 = 2(y+1)$

$y^2 - 2y - 3 = 0$

$(y-3)(y+1) = 0$

$y = 3, -1$

$$\int_{-1}^{3} \left(2(y+1) - (y^2-1)\right) dy$$

$$= \int_{-1}^{3} (-y^2 + 2y + 3)\,dy$$

$$= -\frac{1}{3}y^3 + y^2 + 3y\Big|_{-1}^{3}$$

$= (-9+9+9) - (\frac{1}{3} + 1 - 3)$

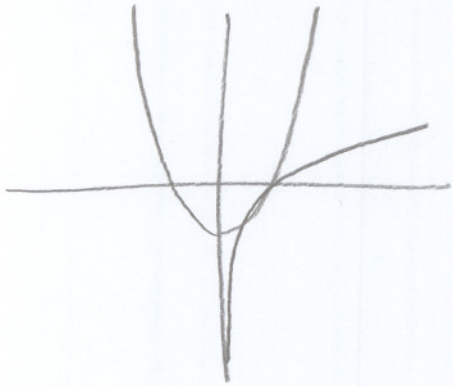
$= 11 - \frac{1}{3}$

$= 10\frac{2}{3}$

1 POINT EACH

[QUESTIONS ON OTHER SIDE]

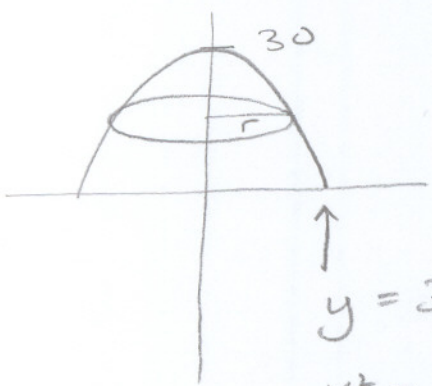
[3 POINTS] Set up an integral to find the area of the region bounded by the curves $y = x^2 - 1$ and $y = \ln x$. Use fnInt to find the area.



$$\int_{.45076365}^{1} (\ln x - (x^2 - 1))\, dx$$

$$= 0.05637705145$$

[6 POINTS] The outline of a dome is given by $y = 30 - \frac{x^2}{120}$ for $-60 \le x \le 60$ (units of feet), with circular cross sections perpendicular to the y-axis. Find its volume exactly **(ie. not a decimal approximation, not using fnInt)**.



$$A(y) = \pi r^2 \text{ WHERE } r = \sqrt{120(30-y)}$$

$$= 120\pi(30-y)$$

$$y = 30 - \frac{x^2}{120}$$

$$x^2 = 120(30-y)$$

$$x = \sqrt{120(30-y)}$$

$$\int_0^{30} 120\pi(30-y)\, dy$$

$$= 120\pi \int_0^{30} (30-y)\, dy$$

$$= 120\pi \left(30y - \tfrac{1}{2}y^2\right)\Big|_0^{30}$$

$$= 120\pi(900 - 450)$$

$$= 54{,}000\pi \text{ ft}^3$$

MUST BE IN THIS FORM — NOT A DECIMAL