Math 1B
Quiz 5
Thu Feb 14, 2008 ♥

$-\frac{1}{2}$ EACH IF YOU FORGET π, 2π, dx OR dy ON ANY PROBLEM

Code: ____________________
I forgot my code, so please charge me two points: (Name) ____________

YOU MAY USE YOUR CALCULATOR TO SKETCH GRAPHS AND FIND INTERSECTIONS, BUT YOU MUST *NOT* USE fnInt

[7 POINTS] Find the exact volume if the region bounded by $x = (y-3)^2$ and $x = 4$ is revolved around the x-axis.
Decimal approximations are NOT acceptable.

SHELL METHOD:

1 2

$$\int_1^5 2\pi y(4-(y-3)^2)\,dy$$

$$= 2\pi\int_1^5 y(4-y^2+6y-9)\,dy$$

1

$$= 2\pi\int_1^5 y(-y^2+6y-5)\,dy$$

OR $-y^3+6y^2-5y$

1

$$= 2\pi\left(-\frac{1}{4}y^4+2y^3-\frac{5}{2}y^2\right)\Bigg|_1^5$$

1

$$= 2\pi\left(-\frac{625}{4}+250-\frac{125}{2}-\left(-\frac{1}{4}+2-\frac{5}{2}\right)\right)$$

1

$$= 64\pi$$

WASHER METHOD:

1 2

$$\int_0^4 \pi\left(\left(3+\sqrt{x}\right)^2-\left(3-\sqrt{x}\right)^2\right)dx$$

$$= \pi\int_0^4\left(9+6\sqrt{x}+x-\left(9-6\sqrt{x}+x\right)\right)dx$$

2

$$= \pi\int_0^4 12\sqrt{x}\,dx$$

$$= 12\pi\int_0^4 x^{\frac{1}{2}}\,dx$$

1

$$= 12\pi\left(\frac{2}{3}x^{\frac{3}{2}}\right)\Bigg|_0^4$$

$$= 12\pi\left(\frac{16}{3}\right)$$

$$= 64\pi$$ 1

[6 POINTS] The base of a solid is the region bounded by $y = 3x+2$ and $y = 6-x$. Cross sections perpendicular to the y-axis are semicircles. Use calculus to find the exact volume of the solid. **Decimal approximations are NOT acceptable.**

CROSS SECTIONS:

IF YOU THOUGHT THE REGION WAS BOUNDED BY THE x-AXIS

1 1

$$A = \frac{1}{2}\pi\left(\frac{1}{2}s\right)^2 = \frac{\pi}{8}s^2 \text{ where } s = 6-y-\frac{y-2}{3} = \frac{18-3y-y+2}{3} = \frac{20-4y}{3} = \frac{4}{3}(5-y)$$

$$\text{So, } A = \frac{\pi}{8}\frac{16}{9}(5-y)^2 = \frac{2\pi}{9}(5-y)^2$$

$$\frac{2\pi}{9}\int_0^5 (5-y)^2\,dy$$

1 1

$$= \frac{2\pi}{9}\int_0^5\left(25-10y+y^2\right)dy$$

1

$$= \frac{2\pi}{9}\left(25y-5y^2+\frac{1}{3}y^3\right)\Bigg|_0^5$$

$$= \frac{2\pi}{9}\left(125-125+\frac{125}{3}\right)$$

$\frac{1}{2}$

$$= \frac{250}{27}\pi$$

$\frac{1}{2}$

IF YOU THOUGHT THE REGION WAS BOUNDED BY THE y-AXIS

$$A = \frac{1}{2}\pi\left(\frac{1}{2}s\right)^2 = \frac{\pi}{8}s^2 \text{ where } s = 6-y \text{ if } y\in[5,6], \text{ or } \frac{y-2}{3} \text{ if } y\in[2,5]$$

1 1

So, $A = \frac{\pi}{8}(6-y)^2$ if $y\in[5,6]$, or $\frac{\pi}{72}(y-2)^2$ if $y\in[2,5]$

1 TOGETHER

1 TOGETHER

$$\frac{\pi}{72}\int_2^5 (y-2)^2\,dy + \frac{\pi}{8}\int_5^6 (6-y)^2\,dy$$

$$= \frac{\pi}{72}\int_2^5 \left(y^2-4y+4\right)dy + \frac{\pi}{8}\int_5^6 \left(36-12y+y^2\right)dy$$

$\frac{1}{2}$ $\frac{1}{2}$

$$= \frac{\pi}{72}\left(\frac{1}{3}y^3-2y^2+4y\right)\Bigg|_2^5 + \frac{\pi}{8}\left(36y-6y^2+\frac{1}{3}y^3\right)\Bigg|_5^6$$

$$= \frac{\pi}{72}\left(\frac{125}{3}-50+20-\left(\frac{8}{3}-8+8\right)\right) + \frac{\pi}{8}\left(216-216+72-\left(180-150+\frac{125}{3}\right)\right)$$

$$= \frac{\pi}{8}+\frac{\pi}{24} \quad \frac{1}{2}$$

$$= \frac{\pi}{6} \quad \frac{1}{2}$$

−1 IF YOU THOUGHT THE REGION WAS BOUNDED BY THE y-AXIS AND YOU ARE IN THE 11:30AM CLASS

(NO PENALTY IF YOU ARE IN THE 8:30AM CLASS)

[7 POINTS] Find the exact volume if the region bounded by $y = x$, $y=\sqrt{6-x}$ and $y = 0$ is revolved around $x = 6$.
Decimal approximations are NOT acceptable.

SHELL METHOD:

1 TOGETHER

$$\int_0^2 2\pi(6-x)x\,dx + \int_2^6 2\pi(6-x)\sqrt{6-x}\,dx$$

2 TOGETHER

$$= 2\pi\left(\int_0^2 \left(6x-x^2\right)dx + \int_2^6 (6-x)^{\frac{3}{2}}\,dx\right)$$

$\frac{1}{2}$ $\frac{1}{2}$

$$= 2\pi\left(\left(3x^2-\frac{1}{3}x^3\right)\Bigg|_0^2 + \left(-\frac{2}{5}(6-x)^{\frac{5}{2}}\right)\Bigg|_2^6\right)$$

$\frac{1}{2}$ $\frac{1}{2}$

$$= 2\pi\left(12-\frac{8}{3}-\frac{2}{5}\left(0-4^{\frac{5}{2}}\right)\right) \quad 1$$

$$= 2\pi\left(\frac{28}{3}+\frac{64}{5}\right)$$

$$= \frac{664\pi}{15} \quad 1$$

WASHER METHOD:

1 2

$$\int_0^2 \pi\left((6-y)^2-\left(6-\left(6-y^2\right)\right)^2\right)dy$$

$$= \pi\int_0^2 \left(36-12y+y^2-y^4\right)dy \quad 1$$

$$= \pi\left(36y-6y^2+\frac{1}{3}y^3-\frac{1}{5}y^5\right)\Bigg|_0^2 \quad 1$$

$$= \pi\left(72-24+\frac{8}{3}-\frac{32}{5}\right) \quad 1$$

$$= \frac{664}{15}\pi \quad 1$$