Math 49B
Midterm 1
Fri Feb 1, 2008

Code: ______________________________
I forgot my code, so please charge me two points: (Name) ______________

[15 POINTS] Sketch the graph of $y = 3\sin\left(\frac{\pi}{2}x + \pi\right) - 5$. Include 2 periods. Label all x- and y-coordinates discussed in class.

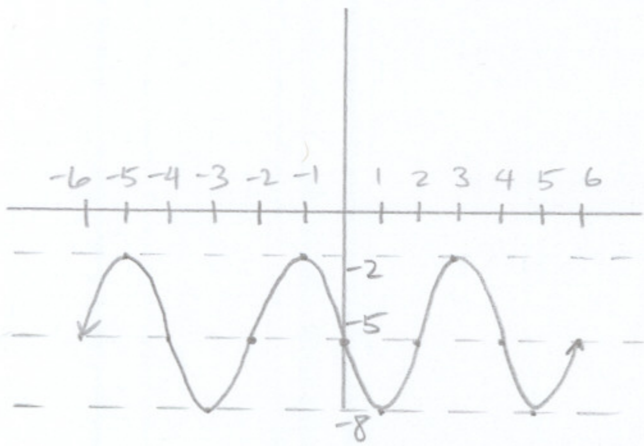
SHOW YOUR WORK.

MIDDLE $y = -5$

AMPLITUDE $|3| = 3$ MAX $= -5 + 3 = -2$ MIN $= -5 - 3 = -8$

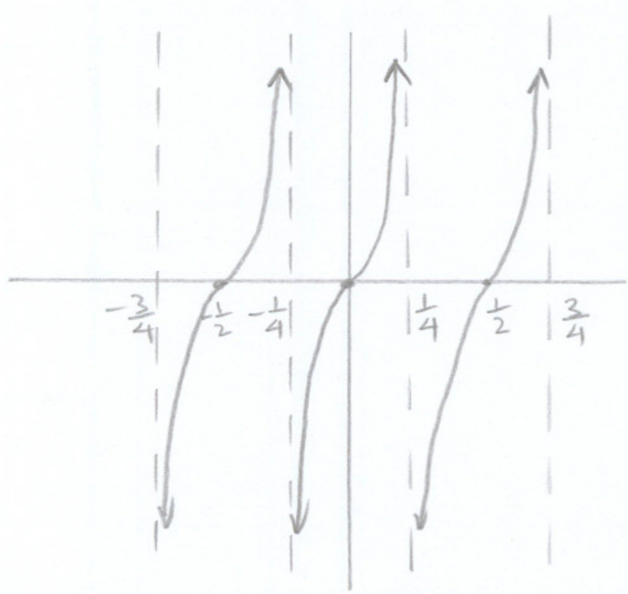
PERIOD $\frac{2\pi}{\frac{\pi}{2}} = 4$ $\frac{1}{4}$ PERIOD $= 1$

"STARTS" AT $\frac{-\pi}{\frac{\pi}{2}} = -2$



[10 POINTS] Sketch the graph of $y = \tan 2\pi x$. Include 2 periods. Label all x-coordinates discussed in class.

PERIOD $= \frac{\pi}{2\pi} = \frac{1}{2}$



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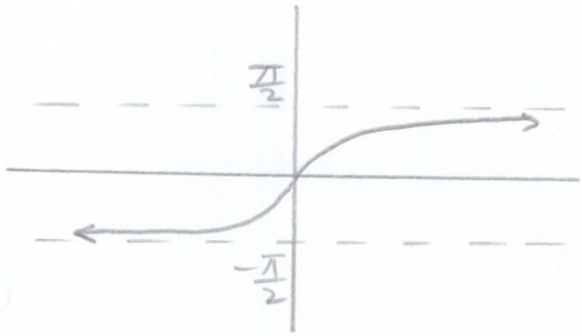
[8 POINTS] Sketch the graph of $y = \tan^{-1} x$, and state the domain and range.

DOMAIN = $(-\infty, \infty)$ RANGE = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



[12 POINTS] Evaluate the following expressions.

(a) $\sin^{-1}(1) = \frac{\pi}{2}$

(b) $\arctan(-\sqrt{3}) = -\frac{\pi}{3}$

(c) $\cos^{-1}(0) = \frac{\pi}{2}$

(d) $\arccos\left(\cos\frac{4\pi}{3}\right) =$

$\arccos\left(-\frac{1}{2}\right)$

$= \frac{2\pi}{3}$

(e) $\tan^{-1}\left(\tan\left(\frac{\pi}{5}\right)\right) = \frac{\pi}{5}$

(f) $\sin(\arcsin(-2)) =$

UNDEFINED

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[12 POINTS] If $\cot x = 3$ and $\csc x < 0$, **use identities** to find the values of $\sec x$ and $\sin x$.

$\csc^2 x = 1 + \cot^2 x$

$\csc^2 x = 10$

$\csc x = -\sqrt{10}$

$\sin x = \dfrac{1}{\csc x} = -\dfrac{1}{\sqrt{10}}$

$\tan x = \dfrac{\sin x}{\cos x}$ SO $\dfrac{\tan x}{\sin x} = \dfrac{1}{\cos x}$ OR $\dfrac{\csc x}{\cot x} = \sec x$

$\sec x = -\dfrac{\sqrt{10}}{3}$

[8 POINTS] Use the trigonometric substitution $x = 3\cot\theta$ to write $\sqrt{4x^2 + 36}$ as a trigonometric function of θ, where $0 < \theta < \dfrac{\pi}{2}$.

$\sqrt{4(3\cot\theta)^2 + 36}$

$= \sqrt{36\cot^2\theta + 36}$

$= \sqrt{36}\sqrt{\cot^2\theta + 1}$

$= 6\sqrt{\csc^2\theta} = 6\csc\theta$

[6 POINTS] Write $\sqrt{\dfrac{1-\cos 80°}{2}}$ as the sine, cosine or tangent of an angle.

$\sin \frac{1}{2}(80°)$

$= \sin 40°$

[10 POINTS] Write an algebraic expression that is equivalent to $\cos(2\cos^{-1} x)$.

LET $\theta = \cos^{-1} x$

SO $x = \cos\theta$

$\cos(2\cos^{-1} x) = \cos 2\theta$

$= 2\cos^2\theta - 1$

$= 2x^2 - 1$

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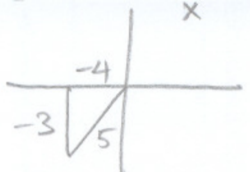

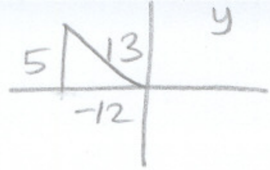

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[20 POINTS] If $\pi < x < \frac{3\pi}{2}$ and $\frac{\pi}{2} < y < \pi$, and $\cos x = -\frac{4}{5}$ and $\cot y = -\frac{12}{5}$, find the value of the following expressions **algebraically**. (You may use your calculator to perform additions, subtractions, multiplications and divisions.)

(a) $\sin 2x = 2\sin x \cos x$

$= 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$= \frac{24}{25}$

(b) $\tan\frac{y}{2} = \frac{1-\cos y}{\sin y}$

$= \frac{1-\left(-\frac{12}{13}\right)}{\frac{5}{13}}$

$= \frac{\overset{5}{\cancel{25}}}{\cancel{13}} \cdot \frac{\cancel{13}}{\cancel{5}}$

$= 5$

(c) $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$= \left(-\frac{4}{5}\right)\left(\frac{-12}{13}\right) + \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right)$

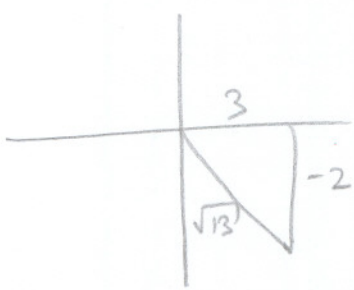
$= \frac{48}{65} - \frac{15}{65}$

$= \frac{33}{65}$

[10 POINTS] Find the exact value of $\cos\left(\tan^{-1}\left(-\frac{2}{3}\right)\right)$.

LET $\theta = \tan^{-1}\left(-\frac{2}{3}\right)$

$\tan\theta = -\frac{2}{3}$ AND $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ SO θ IN Q_4



$\cos\theta = \frac{3}{\sqrt{13}}$

$\cos\left(\tan^{-1}\left(-\frac{2}{3}\right)\right) = \frac{3}{\sqrt{13}}$

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[12 POINTS] Find the exact solutions of the equation $1+2\sin\frac{x}{2}=0$ in the interval $[0, 2\pi)$ **algebraically**.

$\sin\frac{x}{2} = -\frac{1}{2}$

$0 \le x < 2\pi$

$0 \le \frac{x}{2} < \pi$

NO SOLUTION FOR

$0 \le \frac{x}{2} < \pi$

[12 POINTS] Find the exact solutions of the equation $\cos 2x = 1+\sin x$ **algebraically**.

$1-2\sin^2 x = 1+\sin x$

$0 = 2\sin^2 x + \sin x$

$0 = \sin x(2\sin x + 1)$

$\sin x = 0$ OR $\sin x = -\frac{1}{2}$

$x = n\pi$ OR $\frac{7\pi}{6} + 2n\pi$ OR $\frac{11\pi}{6} + 2n\pi$

[15 POINTS] Verify the identity $\frac{\cos x}{1-\sin x} - \frac{1+\sin x}{\cos x} = \sec^2 x - \csc^2 x + \cot^2 x - \tan^2 x$.

$\frac{\cos}{1-\sin x} - \frac{1+\sin x}{\cos x}$

$= \frac{\cos^2 x - (1-\sin x)(1+\sin x)}{(1-\sin x)\cos x}$

$= \frac{\cos^2 x - (1-\sin^2 x)}{(1-\sin x)\cos x}$

$= \frac{\cos^2 x - \cos^2 x}{(1-\sin x)\cos x}$

$= 0$

$\sec^2 x - \csc^2 x + \cot^2 x - \tan^2 x$

$= \cancel{1} + \tan^2 x - \csc^2 x$
$+ \csc^2 x - \cancel{1} - \tan^2 x$

$= 0$

QED