Math 49B
Midterm 2
Wed Feb 27, 2008

Code: _____
I forgot my code, so please charge me two points: (Name) _____

THIS IS A NO GRAPHING CALCULATOR TEST. YOU MAY USE A SCIENTIFIC CALCULATOR.

[9 POINTS] Solve $3\underbrace{\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}}_{A} - 2X = \underbrace{\begin{bmatrix}0 & 5\\ 1 & 3\end{bmatrix}}_{B}$ for X. **SHOW YOUR WORK.**

$3A - 2X = B$

$-2X = -3A + B$

$X = -\frac{1}{2}(-3A + B)$

$X = -\frac{1}{2}\left(3\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix} + \begin{bmatrix}0 & 5\\ 1 & 3\end{bmatrix}\right)$

$X = -\frac{1}{2}\left(\begin{bmatrix}-3 & -12\\ -6 & 0\end{bmatrix} + \begin{bmatrix}0 & 5\\ 1 & 3\end{bmatrix}\right)$

$X = -\frac{1}{2}\begin{bmatrix}-3 & -7\\ -5 & 3\end{bmatrix}$

$X = \begin{bmatrix}1.5 & 3.5\\ 2.5 & -1.5\end{bmatrix}$

[12 POINTS] Write parametric equations for the following.

[a] the circle with a diameter with endpoints (0, 1) and (6, 9)

CENTER $= \left(\frac{0+6}{2}, \frac{1+9}{2}\right) = (3, 5)$

RADIUS $= \sqrt{(3-0)^2 + (5-1)^2} = 5$

$x = 3 + 5\cos\theta$

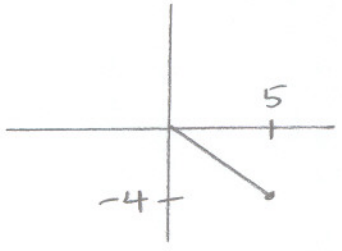
$y = 5 + 5\sin\theta$

[b] the line from (–1, 4) to (–7, 7)

$x = -1 + (-7 - -1)t = -1 - 6t$

$y = 4 + (7 - 4)t = 4 + 3t$

[6 POINTS] Convert the point with rectangular coordinates (5, –4) to polar coordinates.



$r = \sqrt{5^2 + (-4)^2} = \sqrt{41}$

$\theta = \tan^{-1}\left(-\frac{4}{5}\right)$

$\left(\sqrt{41}, \tan^{-1}\left(-\frac{4}{5}\right)\right)$

[12 POINTS] Janice is throwing an Australian themed party for her son Nathan's birthday. Each of Nathan's 30 guests will receive a stuffed animal. Wombats cost \$4 each, platypuses cost \$5 each and Tasmanian devils cost \$7 each. Janice plans to spend exactly \$150 on the toys. Since the wombats are the cheapest, she plans to buy twice as many wombats as Tasmanian devils. Janice would like to know how many of each stuffed animal to buy.

Write **BUT DO NOT SOLVE** an augmented matrix for the problem. State clearly what each unknown represents.

W = # WOMBATS
P = # PLATYPUS
T = # TASMANIAN DEVILS

$W + P + T = 30$

$4W + 5P + 7T = 150$

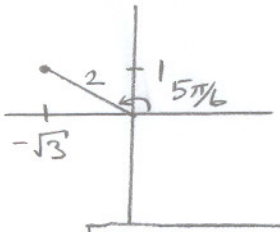
$W = 2T$ OR $W - 2T = 0$

$$\left[\begin{array}{ccc|c}1 & 1 & 1 & 30\\ 4 & 5 & 7 & 150\\ 1 & 0 & -2 & 0\end{array}\right]$$

[12 POINTS] Convert the rectangular equation $x^2 - y^2 = 2x$ to polar form and simplify. **SHOW YOUR WORK.**

$$r^2\cos^2\theta - r^2\sin^2\theta = 2r\cos\theta$$
$$r(\cos^2\theta - \sin^2\theta) = 2\cos\theta$$
$$r\cos 2\theta = 2\cos\theta$$
$$r = \frac{2\cos\theta}{\cos 2\theta}$$

[12 POINTS] Use DeMoivre's Theorem to find the value of $\left(-\sqrt{3}+i\right)^5$. Write the answer in standard form. **SHOW YOUR WORK.**



$r = \sqrt{(-\sqrt{3})^2 + 1^2} = 2$

$\theta = \pi + \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$
$= \pi + \left(-\frac{\pi}{6}\right)$
$= \frac{5\pi}{6}$

$= \left(2\,\mathrm{cis}\,\frac{5\pi}{6}\right)^5$
$= 2^5\,\mathrm{cis}\,\frac{25\pi}{6}$
$= 32\,\mathrm{cis}\,\frac{\pi}{6}$
$= 32\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$
$= 16\sqrt{3} + 16i$

$\frac{25\pi}{6} = 4\pi + \frac{\pi}{6}$

[18 POINTS] Consider the system of linear equations.

$$2x + 5y - z = 5$$
$$3x + 7y - z = 6$$
$$-x - 3y + z = -4$$

Use Gauss-Jordan elimination on the augmented matrix to get a matrix in reduced row echelon form, and solve the system. State the row operations you are performing. (You may use the notation used in class eg. $R_1 \leftarrow R_1 + 2*R_2$).

$$\left[\begin{array}{ccc|c} 2 & 5 & -1 & 5 \\ 3 & 7 & -1 & 6 \\ -1 & -3 & 1 & -4 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} -1 & -3 & 1 & -4 \\ 3 & 7 & -1 & 6 \\ 2 & 5 & -1 & 5 \end{array}\right] R_1 \leftarrow -R_1$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 3 & 7 & -1 & 6 \\ 2 & 5 & -1 & 5 \end{array}\right] \begin{array}{l} R_2 \leftarrow R_2 + (-3)R_1 \\ R_3 \leftarrow R_3 + (-2)R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 0 & -2 & 2 & -6 \\ 0 & -1 & 1 & -3 \end{array}\right] R_2 \leftarrow -\tfrac{1}{2}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & -1 & 1 & -3 \end{array}\right] R_3 \leftarrow R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] R_1 \leftarrow R_1 + (-3)R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + 2z = -5$ $\quad x = -2z - 5$
$y - z = 3$ $\quad y = z + 3$
$z = z$

$(-2z-5, z+3, z)$

[27 POINTS] Consider the graph of the polar equation $r = 1 - 2\cos 2\theta$.

[a] Is the graph symmetric with respect to the polar axis ? **SHOW YOUR WORK.**

$r = 1 - 2\cos 2(-\theta)$

$r = 1 - 2\cos(-2\theta)$

$r = 1 - 2\cos 2\theta$ YES

[b] Is the graph symmetric with respect to the pole ? **SHOW YOUR WORK.**

$r = 1 - 2\cos 2(\pi + \theta)$

$r = 1 - 2\cos(2\pi + 2\theta)$

$r = 1 - 2(\cos 2\pi \cos 2\theta - \sin 2\pi \sin 2\theta)$

$r = 1 - 2\cos 2\theta$ YES

[c] Is the graph symmetric with respect to $\theta = \frac{\pi}{2}$?

YES, SINCE IT'S ALREADY SYMMETRIC THE OTHER 2 WAYS

[d] Find the values of θ for which r has its maximum and minimum values. **SHOW YOUR WORK.**

MAX $r = 1 + 2 = 3$

MIN $r = 1 - 2 = -1$

$1 - 2\cos 2\theta = 3$

$\cos 2\theta = -1$

$2\theta = \pi, 3\pi$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$0 \le \theta < 2\pi$

$0 \le 2\theta < 4\pi$

$1 - 2\cos 2\theta = -1$

$\cos 2\theta = 1$

$2\theta = 0, 2\pi$

$\theta = 0, \pi$

[e] Find the values of θ for which the graph passes through the pole. **SHOW YOUR WORK.**

$1 - 2\cos 2\theta = 0$

$\cos 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

[27 POINTS] Consider the conic with the polar equation $r = \dfrac{5}{2-3\sin\theta}$.

[a] Find the eccentricity. **SHOW YOUR WORK.**

$\dfrac{5}{2-3\sin\theta} \cdot \dfrac{\frac{1}{2}}{\frac{1}{2}} = \dfrac{\frac{5}{2}}{1-\frac{3}{2}\sin\theta}$ $\quad e = \frac{3}{2}$

[b] Find the equation of the directrix in rectangular form. **SHOW YOUR WORK.**

$ep = \frac{5}{2}$

$\frac{3}{2}p = \frac{5}{2}$

$p = \frac{5}{3}$, HORIZONTAL, BELOW $\quad y = -\frac{5}{3}$

[c] Identify the type of conic.

HYPERBOLA

[d] Find both foci in rectangular coordinates. **SHOW YOUR WORK.**

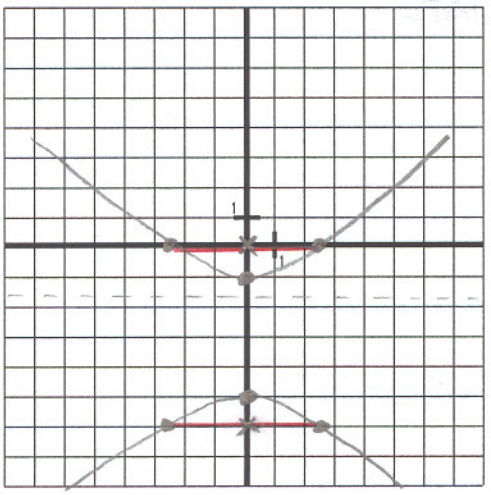
θ	r
0	5/2
$\pi/2$	-5
π	5/2
$3\pi/2$	1

CENTER $= \frac{-1+-5}{2} = -3$

FOCUS $= 2(-3) = -6$

FOCI (0,0) (0,-6)

[e] Sketch the conic with both latera recta.



[f] Write the equation in rectangular form and simplify. **SHOW YOUR WORK.**

$2r - 3r\sin\theta = 5$

$2\sqrt{x^2+y^2} - 3y = 5$

$2\sqrt{x^2+y^2} = 3y + 5$

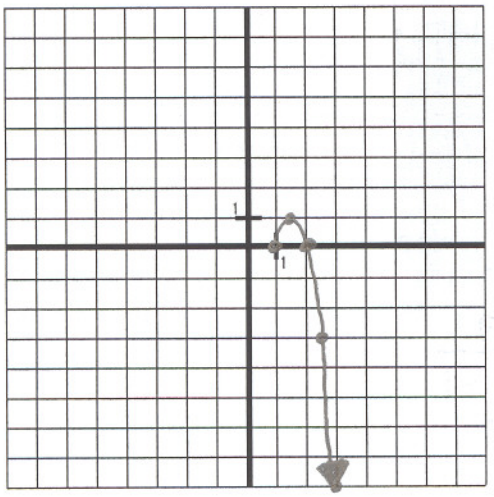
$4(x^2+y^2) = 9y^2 + 30y + 25$

$4x^2 + 4y^2 = 9y^2 + 30y + 25$

$4x^2 - 5y^2 - 30y - 25 = 0$

[15 POINTS] Consider the parametric equations $x = \frac{1}{2}t + 1$, $y = 2t - t^2$ for $0 \le t \le 4$.

[a] Sketch the curve represented by the equations (including the orientation).



t	x	y
0	1	0
1	1.5	1
2	2	0
3	2.5	-3
4	3	-8

[b] Write the corresponding rectangular equation. Simplify your answer. **SHOW YOUR WORK.**

$$t = 2(x-1)$$
$$y = 2(2(x-1)) - (2(x-1))^2$$
$$= 4(x-1) - 4(x-1)^2$$
$$= 4(x-1)(1-(x-1))$$
$$= 4(x-1)(2-x)$$
$$= 4(-x^2+3x-2)$$
$$= -4x^2+12x-8$$

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In class and in the textbook, you learned how to test if polar graphs were symmetric in various ways.

[4 BONUS POINTS] Find 2 tests that can be used to determine if a graph is symmetric over the line $\theta = \frac{\pi}{4}$.

[6 BONUS POINTS] Find 2 tests that can be used to determine if a graph is symmetric over the line $\theta = \theta_0$ for any arbitrary θ_0.