Math 49B
Midterm 3
Tue Mar 18, 2008

Code: ______

THIS IS A NO GRAPHING CALCULATOR TEST. YOU MAY USE A SCIENTIFIC CALCULATOR.

[redacted]

Let A be the point (1, –2, 0).
Let B be the point (–1, 0, 2).
Let C be the point (0, 2, –1).

[redacted]

Let **u** be the vector with initial point A and terminal point B.
Let **v** be the vector with initial point A and terminal point C.

[3 POINTS] Find the component forms of **u** and **v**.

$\vec{u} = \langle -1-1,\ 0-{}^-2,\ 2-0\rangle = \langle -2, +2, 2\rangle$

$\vec{v} = \langle 0-1,\ 2-{}^-2,\ -1-0\rangle = \langle -1, 4, -1\rangle$

[6 POINTS] Determine if **u** and **v** are parallel.

$\langle -2, +2, 2\rangle = k\langle -1, 4, -1\rangle$

$-2 = -k$
$+2 = 4k$
$2 = -k$

IMPOSSIBLE, NOT PARALLEL

[8 POINTS] Find the angle between **u** and **v**.

$\theta = \cos^{-1} \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}$

$= \cos^{-1} \dfrac{(-2)(-1)+(+2)(4)+(2)(-1)}{\sqrt{(-2)^2+(-2)^2+2^2}\ \sqrt{(-1)^2+4^2+(-1)^2}}$

$= \cos^{-1} \dfrac{+8}{\sqrt{12}\sqrt{18}}$

$= \cancel{122.98^\circ}\ 57.02^\circ$

[6 POINTS] Find a vector of magnitude 5 in the same direction as **v**.

$\dfrac{5}{\|\vec{v}\|}\vec{v} = \dfrac{5}{\sqrt{18}}\langle -1, 4, -1\rangle$

$= \dfrac{5\sqrt{2}}{6}\langle -1, 4, -1\rangle$

$= \left\langle -\dfrac{5\sqrt{2}}{6}, \dfrac{10\sqrt{2}}{3}, -\dfrac{5\sqrt{2}}{6}\right\rangle$

[9 POINTS] Find the projection of **u** onto **v**.

$$\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{v} = \frac{+8}{18}\langle -1, 4, -1\rangle$$

$$= \left\langle -\frac{4}{9}, +\frac{16}{9}, -\frac{4}{9}\right\rangle$$

[12 POINTS] Find a unit vector that is perpendicular to both **u** and **v**.

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & +2 & 2 \\ -1 & 4 & -1 \end{vmatrix} = (-2-8)\vec{i} - (2--2)\vec{j} + (-8--2)\vec{k}$$

$$= \langle \cancel{-6}, -4, \cancel{-10}\rangle$$

$$\quad -10 \qquad\quad -6$$

[6 POINTS] Find the area of the triangle ABC.

$$\frac{1}{2}\|\vec{u}\times\vec{v}\| = \frac{1}{2}\sqrt{(-6)^2+(-4)^2+(-10)^2}$$

$$= \frac{1}{2}\sqrt{152}$$

$$= \sqrt{38}$$

[12 POINTS] Find the equation of the sphere which has A and B as endpoints of a diameter.

$$\text{CENTER} = \left(\frac{1+(-1)}{2}, \frac{-2+0}{2}, \frac{0+2}{2}\right) = (0, -1, 1)$$

$$\text{RADIUS} = \sqrt{(0-1)^2+(-1--2)^2+(1-0)^2} \quad \text{OR } \frac{1}{2}\|\vec{u}\| = \sqrt{3}$$

$$= \sqrt{3}$$

$$(x-0)^2+(y--1)^2+(z-1)^2 = \sqrt{3}^2$$

$$x^2+(y+1)^2+(z-1)^2 = 3$$

[6 POINTS] Find the point in 3-space which is 2 units left of the xz-plane, 5 units below the xy-plane and 6 units in front of the yz-plane.

$(6, -2, -5)$

[20 POINTS] Let $\mathbf{A} = \begin{bmatrix} \overset{+}{-3} & -9 & 5 & -1 \\ \overset{-}{6} & -7 & -8 & 4 \\ \overset{+}{0} & 2 & 0 & 0 \\ 1 & -3 & 0 & 2 \end{bmatrix}$.

[a] Find $|\mathbf{A}|$.

$$-2 \begin{vmatrix} \overset{+}{-3} & 5 & -1 \\ \overset{-}{6} & -8 & 4 \\ \overset{+}{1} & \overset{-}{0} & \overset{+}{2} \end{vmatrix}$$

$$= -2 \left(1 \begin{vmatrix} 5 & -1 \\ -8 & 4 \end{vmatrix} + 2 \begin{vmatrix} -3 & 5 \\ 6 & -8 \end{vmatrix} \right)$$

$$= -2 \left((20-8) + 2(24-30) \right)$$

$$= -2 (12 - 12) = 0$$

[b] Does $\mathbf{A}^{-1}$ exist ? Why or why not ?

NO, $|A| = 0$.

[15 POINTS] Use an inverse matrix to find the solution of $\begin{aligned} 7x + 9y &= 11 \\ 8x + 7y &= -17 \end{aligned}$.

$$\begin{bmatrix} 7 & 9 \\ 8 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ -17 \end{bmatrix}$$

$$A \quad X = B$$

$$X = A^{-1}B$$

$$= \frac{1}{49-72} \begin{bmatrix} 7 & -9 \\ -8 & 7 \end{bmatrix} \begin{bmatrix} 11 \\ -17 \end{bmatrix}$$

$$= \frac{1}{-23} \begin{bmatrix} 77 + 153 \\ -88 - 119 \end{bmatrix}$$

$$= -\frac{1}{23} \begin{bmatrix} 230 \\ -207 \end{bmatrix}$$

$$= \begin{bmatrix} -10 \\ 9 \end{bmatrix}$$

$x = -10$
$y = 9$

3×4 $\quad$ 2×4 $\quad$ 2×3

[12 POINTS] Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$, and $\mathbf{C} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

Which one of the products **AB**, **AC**, **BA**, **BC**, **CA** or **CB** exists, and what is its value ?

$$\underset{2\times 3}{C}\underset{3\times 4}{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 1 \end{bmatrix}$$

[20 POINTS] Find the partial fraction decomposition of $\dfrac{x^3 - 2}{x^3 - x^2 - 2x}$.

$$\begin{array}{r} 1 \\ x^3 - x^2 - 2x \,\big)\, \overline{x^3 \qquad\qquad -2} \\ \underline{x^3 - x^2 - 2x} \\ x^2 + 2x - 2 \end{array}$$

$$1 + \frac{x^2 + 2x - 2}{x^3 - x^2 - 2x}$$

$$= 1 + \frac{x^2 + 2x - 2}{x(x^2 - x - 2)}$$

$$= 1 + \frac{x^2 + 2x - 2}{x(x+1)(x-2)}$$

$$\frac{x^2 + 2x - 2}{x(x+1)(x-2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-2}$$

$$x^2 + 2x - 2 = A(x+1)(x-2) + Bx(x-2) + Cx(x+1)$$

$x = 0$: $-2 = -2A$ $\quad A = 1$

$x = -1$: $-3 = 3B$ $\quad B = -1$

$x = 2$: $6 = 6C$ $\quad C = 1$

$$1 + \frac{1}{x} - \frac{1}{x+1} + \frac{1}{x-2}$$

[15 POINTS] Sketch a graph of the solution set of

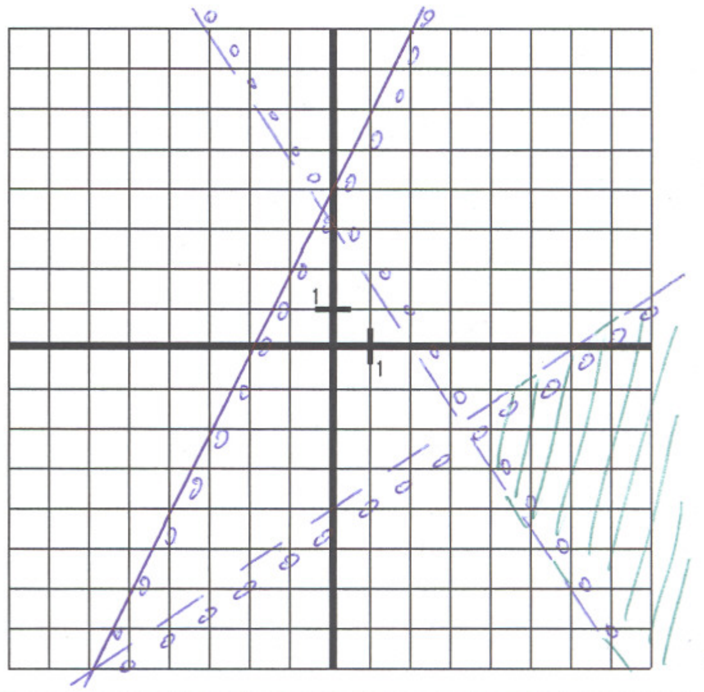
$$\begin{cases} 2x-3y>12 \longrightarrow \text{X-INT} = 6 \quad \text{y-INT} = -4 \quad \text{DOTTED} \\ y-2x \le 4 \longrightarrow \text{X-INT} = -2 \quad \text{y-INT} = 4 \quad \text{SOLID} \\ 3x+2y>6 \longrightarrow \text{X-INT} = 2 \quad \text{y-INT} = 3 \quad \text{DOTTED} \end{cases}$$

TEST (0,0)

$2(0)-3(0)>12$ NO

$0-2(0)\le 4$ YES

$3(0)+2(0)>6$ NO



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[10 BONUS POINTS] Prove that, for all vectors **u** and **v** in 3-space, **u** × **v** is orthogonal to both **u** and **v**.
HINT: Let **u** = u_1**i** + u_2**j** + u_3**k** and **v** = v_1**i** + v_2**j** + v_3**k**.